Cyclical Price Volatility: Role of Shopping Behavior and Customer Capital *

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Abstract

Dispersion in price growth rates rises during recessions. In this paper, I explain this empirical phenomenon from the perspective of consumer shopping behavior and sellers' customer accumulation. I document the facts that (1) consumers switch more across sellers during a recession, and (2) sellers set higher prices after a growth in the customer base. During a downturn, faster switching of consumers results in a larger growth in the customer base of cheap sellers, and they respond by increasing price mark-ups more. The opposite happens to expensive sellers so that the price growth dispersion gets larger. I build a general equilibrium model in which firms accumulate customer capital, and households endogenously decide search effort. When calibrated to match the moments from shopping behavior and cross-sectional distribution of price and customer base, the model explains 30% of the rise in price growth dispersion during the Great Recession.

JEL codes: D21; E31, E32; L11

Keywords: Countercyclical Dispersion, Firm Dynamics, Product Market Frictions, Customer Capital.

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1 Introduction

Dispersion in good-level price growth rates rises during recessions. As shown in Figure 1, the cross-sectional standard deviation and interquartile range of good-level price growth rates both increase significantly during the Great Recession. In this paper, I explain this empirical phenomenon in a frictional goods market where consumers search for prices and firms accumulate customers.

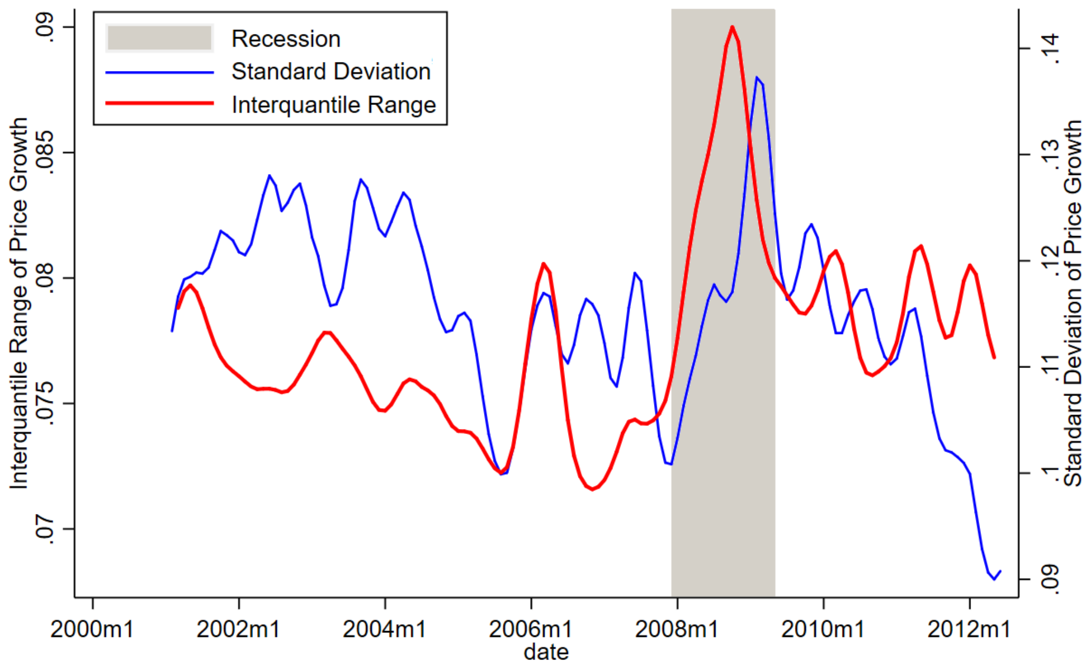
One possible type of explanation is that the rise in dispersion is a result of increased dispersion of exogenous shocks. In models such as Bloom et al. (2012) and Vavra (2014), firms draw exogenous idiosyncratic productivity shocks with time-varying standard deviations, i.e., second-moment shocks. An alternative explanation is that first moment shocks, e.g., shocks to aggregate productivity, induce time-varying behavior of agents, which then results in the variation of the observed cross-sectional second moments.[1] Empirical evidence on the direction of the causality is mixed. Idiosyncratic productivity is measured as revenue TFP, which makes it difficult to disentangle exogenous variations of productivity and endogenous responses of prices. This issue is very important in understanding the mechanisms that drive the business cycle, as well as the related welfare and policy issues. While there are many explanations of counter-cyclical dispersion of firm-level growth rates, little effort has been paid to relate it to the price search behavior of households. In this paper, I provide new empirical evidence that supports a novel mechanism. The mechanism links price change dynamics to consumer shopping behavior and generates counter-cyclical price growth dispersion with only first-moment productivity shock.

I document two facts from micro-level panel data of household transactions. First, I investigate shopping behavior. While most existing empirical work measures total shopping intensity, such as shopping time and number of shopping trips, I decompose shopping trips of households into different margins based on the seller visited in each trip.[2] I focus on the margin that indicates how frequently households switch across sellers. I find that consumers form long-term relations with sellers, and they switch more frequently across sellers during recessions. Second, I identify a positive relationship between the price level set by a seller and the size of its customer base.

[1] See Bachmann and Moscarini (2012), Berger, Dew-Becker and Giglio (2017) and Berger and Vavra (2017).

[2] I consider sellers as units that have direct contact with consumers. In the empirical part, a seller is either a specific store or a specific retailer in a market.

Figure 1: **Dynamics of Price Growth Rate Dispersion**



Notes: The figure plots the average standard deviation and the interquartile range of price growth rate distribution at a monthly frequency. Standard deviations and interquartile ranges are calculated for each geographical market monthly, and then averaged over markets for each month. Detailed data and measurement are discussed in section 2.

The positive relationship is also found between the price level and other measures related to the customer base, such as sales and the number of shopping trips. This finding suggests that a seller's price-setting is related to its customer base, which is in the same spirit of recent work showing that demand variation is important in explaining firm dynamics.[3]

The facts motivate a novel mechanism, in which price growth distribution is affected by time-varying consumer behavior. During recessions, consumers search more for prices and switch faster from relatively expensive sellers to cheap sellers. Hence, the customer base grows more for cheap sellers and declines more for expensive sellers. As a result, the size growth distribution of sellers is more dispersed. As each seller sets a higher price after a growth in its customer base, the counter-cyclical dispersion in size growth rates is converted into the counter-cyclical dispersion of price growth rates.

To study the mechanism quantitatively, I build a general equilibrium model in which firms

[3]See Foster et al. (2008, 2016), Peters (2016) and Hottman et al. (2016).

accumulate customers and households endogenously decide search effort. In the model, each household is attached to a seller. They observe a posted price from the attached seller and choose the search effort for obtaining lower prices. Sellers set prices based on a tradeoff: on the one hand, lower prices reduce the profit per customer; on the other hand, lower price attracts more customers, so that more profit could be made in the future. I use the model to answer the following question: How much increased dispersion in price growth rates during recessions can be explained by the time-varying shopping behavior?

I calibrate the model to match the empirical moments from shopping intensity and cross-sectional distribution of price and customer base. The main quantitative exercise in this paper is to study the transition path of the economy after an unexpected productivity shock. In a Great Recession experiment, when the model is shocked by the empirical TFP series estimated in Fernald (2014), the dispersion of price growth rates increased by 6.1%, which is about 30% of the size in the data.

I show that the model has welfare implications, which are different from the case when the increased dispersion is a result of uncertainty shocks. As households switch to more productive firms during a recession, the production reallocates to more productive firms. This increases efficiency and reduces the welfare loss from the decline in aggregate TFP. Since households are attached to firms hence do not switch back after the recession, the welfare gain does not vanish after the recession.

The model also has implications on firm dynamics that are consistent with recent empirical findings. In the model, small sellers are more responsive to aggregate shocks than large sellers, which is in line with recent findings that cyclicality of firm-level variables declines with firm size.[4] The model generates heterogeneous responses over size because sellers of different sizes face demand with different elasticities. During recessions, the increased price search effort of consumers intensifies the competition between sellers. On average, large sellers are cheaper in the data as well as in the model. Since a household searches more when its affiliated seller gets more expensive, larger share of consumers are captive for large sellers. Hence, large sellers are hurt less by the increased competition that arises from the greater search intensity of consumers during reces-

[4] See Hong (2017) for mark-up, Crouzet and Mehrotra (2017) for sales and investment, and Clymo and Rozsypal (2019) for employment and turnover.

sions. Moreover, the model generates counter-cyclical dispersion of sales growth rates, which is an empirical fact documented in Davis at el. (2007) and Bloom at el. (2012).

Related Literature This paper is mainly related to three strands of the literature. The first is the literature that studies the counter-cyclical dispersion of economic variables such as price change, sales productivity, and unemployment.[5] Bloom et al. (2012) and Vavra (2014) explain the rise in dispersion as a result of increased dispersion of firm's idiosyncratic TFP shocks. The change in the volatility of exogenous shocks is considered as a driving force of the business cycle. On the other hand, change in dispersion is viewed as a result of time-varying responses of firms to the same shocks. Bachmann and Moscarini (2012), Ilut et al. (2018), Baley and Blanco (2019), Decker and D'Erasmo (2018), Munro (2018) and many others study firm's time-varying response using various mechanisms. However, little attention has been paid to mechanisms related to shopping behavior and buyer-seller relations.[6] The key contribution of this paper is to understand the roles of shopping behavior and customer base in shaping the distribution of price change among heterogeneous firms over the business cycle.

My work is also related to literature that studies the role of frictional goods markets in business cycle fluctuations. Aguiar and Hurst (2013), Coibion et al. (2015), Kaplan and Menzio (2015), Nevo and Wong (2016) and Petrosky-Nadeau et al. (2016) document that household shopping behavior changes systematically over the business cycle. Bai et al. (2012) analyze a demand-driven business cycle model where preference shocks affect consumer search incentives and consumption. Kaplan and Menzio (2016) study the interaction of labor and product market frictions that links unemployment dynamics to consumer search effort. However, these papers do not discuss the implications for aggregate dynamics when the buyers form a repeated-purchase relationship with sellers. This paper studies the role of customer base, which turns out to be important for price-setting on the firm side. This suggests that the way buyers and sellers interact is crucial in understanding the aggregate implications of frictional goods markets.

This paper is also in line with literature that studies the link between firm's pricing deci-

[5] See Bloom (2009) (sales growth), Bloom et al. (2012) (revenue TFP and employment growth), and Vavra (2014) (prices).

[6] The closest paper to mine is Munro (2018), which, to my knowledge, is the only paper that relates consumer behavior to counter-cyclical dispersion at the firm level. My work differs as I introduce customer base concerns for the sellers, and thus studies the heterogeneities among sellers of different size.

sions and the customer base. Early papers by Bils (1989), and Rotemberg and Woodford (1991, 1999) analyze pricing behavior under customer retention concerns. More recent studies, such as Kleshchelski and Vincent (2009), Nakamura and Steinsson (2011), and Gourio and Rudanko (2014) provide different reasons for long term customer relationship. While most of the literature uses exogenous reduced-form formulations for customer base movement, I contribute by providing an approach that rationalizes customer base using household search behavior.

The rest of the paper is organized as follows. Section 2 presents the data and facts on shopping behavior and price dynamics. Section 3 describes the model. Section 4 presents some analytical results of the model. Section 5 discusses the calibration strategy. Section 6 presents the quantitative results. Section 7 concludes.

2 Empirical Facts

2.1 Data Description

I use the IRI marketing dataset.[7] It consists of a consumer panel with transaction-level data of each participating households, and a scanner data that includes the good-level sales and quantity for each participating store. The dataset spans a period of 12 years from the first week of January 2001 to the last week of December 2012.

The scanner data contains price and quantity information for retail stores over 50 geographic markets in the U.S. The markets are mostly consistent with Metropolitan Statistical Areas (MSAs), with two of the metropolitan areas are smaller than usual (Eau Claire, WI and Pittsfield, MA). Each retailer outlet reports the weekly sales in dollars and quantity for each good identified by UPC. The dataset contains over 3,500 stores that belong to 138 supply chains.

The consumer panel includes a panel of more than 5,000 households in two of the metropolitan areas in which households provided detailed information on their characteristics and purchases. These characteristics include income level, age, sex, race, employment, geographical market, and

[7] See Bronnenberg et al. (2008) for a detailed description of the data. I would like to thank Information Resources Inc. for making the data available. All estimates and analyses in this paper based on Information Resources Inc. data are by the author and not by Information Resources Inc.

household size. Households who report little transactions are filtered out from the dataset. Households in the panel record information about each of their transactions, including the timing of shopping trip, the store visited, the UPC of the good purchased, total price and quantity. For example, in one transaction, the dataset observes the price and quantity of 12oz cans of soda of a given brand, that brought by a household at a specific store in Pittsfield in January 2012. Participating households enter the information in one of two ways: if the transaction is in one of the participating stores of the dataset, the household shows a card at the store. Otherwise, the household uses a device provided to scan the purchased items.

2.2 Sample Selection

I define a buyer as a household and a seller as a store in the baseline empirical analysis. I define a seller alternatively as the stores belong to the same retailer in a market for robustness. When defined as a specific store in a given market, there are 3,584 different sellers in all years, and each market contains 72 stores on average. At the retail chain level, there are 2,828 pairs of market and chain combinations in all years, and on average, each market has stores that belong to 50 different chains.

I impose several sample selection criteria in my analysis. First, in capturing the price change distributions, I exclude observations of a good if the number of observations of the good in a given market and month is less than 10.[8] Second, to avoid the possible influence of a small number of outliers of prices, I drop observations if the price is more than ten times the average price of the good in the same month and geographical market. Third, for the consumer panel, while the dataset filters out households who report little transactions by reporting frequency, I still observe a small number of households who report little purchases relative to annual income.[9] To avoid the potential bias in shopping behavior measures from these households, I limit my analysis to households whose annual purchase in the data is no less than 10% of its annual income.[10] Lastly, in the consumer panel, to control for the potential structure change of households caused by attrition, I only include households that participate in the panel in at least 3 consecutive years. Table A1 in

[8]For robustness, I use thresholds between 5 to 20, and the results are similar.

[9]There are about 4% households whose annual purchase reported over its income are less than 5%.

[10]I also use thresholds from 5% to 30%. Results are similar.

appendix summarizes the data after sample selection.

2.3 Price Growth Rate Distributions

I use the scanner data to track the good level price change distribution. For a record of good i at seller s in market m and month t, let $TS^s_{i,m,t}$ and $q^s_{i,m,t}$ denote the total sales in dollars and quantity respectively.[11] Then for seller s, the unit price of each good is computed as

$$p^s_{i,m,t} = \frac{TS^s_{i,m,t}}{q^s_{i,m,t}} \,.$$

The price growth rate for good i at given seller s, $G^s_{i,m,t}$, is then calculated as log difference between prices of two adjacent periods, i.e.,

$$G^s_{i,m,t} = log(p^s_{i,m,t}) - log(p^s_{i,m,t-1}) \,.$$

Figure 2 plots the histogram of price growth rates for all goods, sellers and markets in January 2005.[12] The distribution of price growth rates is symmetric and has extra kurtosis compared to a normal distribution. For comparison, the same histogram is plotted for January 2009, a month during the Great Recession. The distribution of price growth rates is still symmetric, but more dispersed compared to January 2005.

I measure the dispersion of price growth rates as standard deviations of $G^s_{i,m,t}$ for all goods in all sellers for each market and time period. Price growth rate of each good seller pair is weighted by the share of sales in the given market and time period, such that

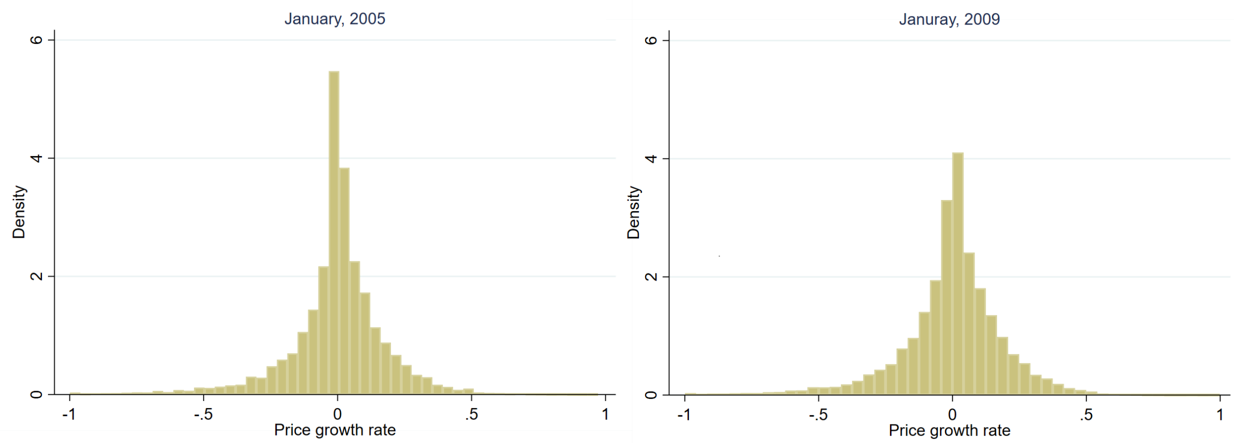
$$\sigma^2_{m,t} = \sum_{i,s} (G^s_{i,m,t} - \bar{G}^s_{i,m,t})^2 \frac{TS^s_{i,m,t}}{\sum_{i,s} TS^s_{i,m,t}} \,.$$

I then regress $\sigma_{m,t}$ on a dummy indicate the Great Recession periods, while controlling for geographical variation and seasonality. During the 2008 recession, the standard deviation of price growth rates increased on average by 0.015 in absolute value, and this is about 12% increase in

[11] A promotion is defined in the data set as a temporary price reduction of more than 5%. I use sellers regular prices, so the deals and discounts are excluded.

[12] Zeros are excluded in the plot and rest of calculations in this section.

Figure 2: **Price Growth Rates Distribution**



Notes: The figure plots the histogram of price growth rates for all goods, sellers and markets in January 2005 and in January 2009. Price growth observations are not weighted.

percentage. Slightly larger numbers are found for the interquartile range measure. This result is robust to the weights used in computing the price growth dispersion at the market level. I calculate the two dispersion measures in each market alternatively by weighting each observation equally and still find an increase in the dispersion of price growth rates during the Great Recession. The detailed regression results are reported in Table A2 in the appendix.

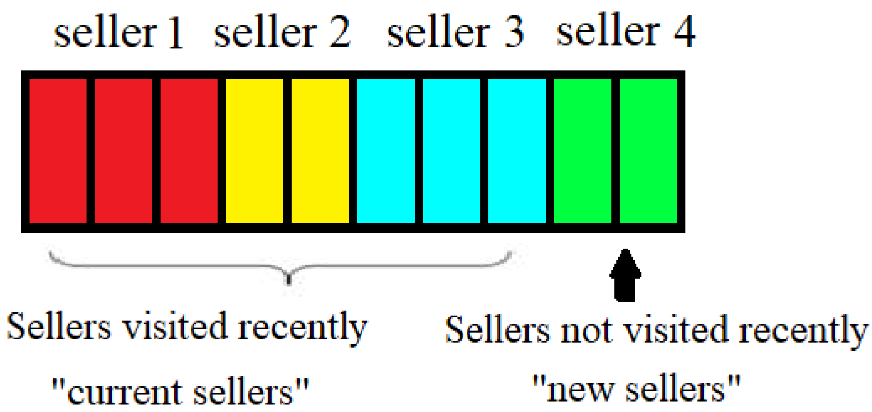
2.4 Shopping Behavior

I investigate the shopping behavior using the shopping trip data in the consumer panel. In the dataset, a household on average takes about 10 shopping trips per month. These shopping trips consist of two parts: (1) the number of shopping trips to the same seller, and (2) the number of different sellers visited. The former indicates how hard a household shops at a given seller, referred to as the intensive margin, and the latter tells how widely a household searches for prices, referred to as the extensive margin. I further decompose sellers into two groups, those that the household had visited recently, referred to as current sellers, and those not visited recently, referred to as new sellers.

Figure 3 illustrates the composition of shopping trips of a typical household during a month. Each grid represents a shopping trip, and the color indicates the seller visited. In the 10 shopping trips in a month, the example household shops from 4 different sellers, and each seller is visited

2.5 times on average. Among the sellers, 3 were current sellers, and one (green) was a new seller.

Figure 3: **An Illustrative Example of Shopping Trips of a Typical Household**



To identify the two components of the extensive margin of shopping, I list the sellers visited within the last n months for each household in each period. I use n=3 in the baseline results shown in this section, I also use n={4,6,9,12}, and results are similar. I calculate the number of different sellers visited that are on the list each month and count the number of shopping trips taken. These numbers are then taken averages over months in a year and then over households. The results are shown in Table 4. Figure 6 plots the distribution of these numbers.

As my baseline in Table 1, I include households whose annual purchase is more than 10% of its annual income. On average, a household takes 10 shopping trips each month. In these trips, 3.6 shopping trips are taken to each store, and around 3 different stores are visited. Among different stores, the average number of new stores visited is 0.44, which is about 14.6% of the total number of different stores. The numbers in the brackets are standard deviations over households. The pooled histogram of shopping behavior in 3 margins are shown by Figure A1 in the appendix. Most households take about 5 to 15 shopping trips per month and visited 2 to 5 different stores. Within extensive margin, in each month, about 25% of households find one new store, and about 10% of households find two or more.

The small fraction of new stores visiting suggests that consumers are "attached" to sellers. Table 1 also shows the results at the retailer level, that the numbers are very close to those for stores. This is because in the dataset, the case that a consumer visit more than one local store that belongs to same supply chain occurs occasionally. The empirical results are robust to different

Table 1: **Summary Statistics of Shopping Intensity Margins**

	Monthly average number of				
	shopping trips	shopping trips per seller	different sellers	current sellers	new sellers
(1)	9.98	3.60	3.02	2.58	0.44
	(7.79)	(2.63)	(1.83)	(1.60)	(0.76)
(2)	9.98	3.61	3.03	2.59	0.44
	(7.79)	(2.63)	(1.82)	(1.61)	(0.76)

Notes: Numbers are calculated by averaging over households in a month and then averaging over months in year 2011. Numbers in the brackets are standard deviations across households. Current sellers are identified as sellers visited in the last 3 months. Sellers are identified as (1) stores (2) retailers in a market.

sample criteria, as the detailed results reported in Table A3 in the appendix.

To study how the shopping intensity varies over time, I regress the log of each of the shopping measures on a recession dummy, control for household demographics and market fix effects. I estimate the following regression:

$$log(N^{new}_{j,m,t}) = \alpha + \beta_1 D^{recession}_t + \gamma D_{j,m,t} + \gamma_2 D_{month} + \epsilon_{j,m,t} \ ,$$

where j indexes for household, m for the geographical market. D is a vector of household demographic variables, which includes income group, age, education, sex, race, household composition, and geographical market. D_{month} are month dummies that adjust for seasonality. The dependent variable for regression is the log of the number of new sellers found by a household. The regression is also estimated using dependent variables include the number of different sellers visited and the number of shopping trips taken.

The regression results are reported in Table 2. During the last recession, the number of different stores a household visit in a month increases by about 3%. This implies that households, on average, search more widely across stores during the Great Recession. However, a big part of the increase is from the number of new sellers. The number of new sellers visited increased by 9.97%, which is a lot bigger than the increase in the extensive margin, suggesting that households

Table 2: **Shopping Intensity Over Time**

		shopping intensity margins		
	(1) total shopping trips	(2) trips per seller	(3) different sellers	(4) new sellers
$D^{recession}$	0.0061**	-0.0189***	0.0305***	0.0997***
	(0.0026)	(0.0023)	(0.0027)	(0.0086)
Geographical FE	✓	✓	✓	✓
Demographics FE	✓	✓	✓	✓
Month FE	✓	✓	✓	✓
No. of Obs.	520,923	520,923	520,923	520,923
R^2	0.0766	0.0671	0.0791	0.655

Notes: The table reports the regression result of each of the log shopping intensity measures on a recession dummy. Each regression controls for the market fixed effect and a set of household demographics, including income group, age, sex, race, education and household composition. Robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

are switching across different sellers more frequently in recessions.

Given households switching more during a recession, what kind of sellers are they switching to? One answer is that households shop at low-priced sellers more because, on average, the income gets lower during a recession. In order to check if this is the case, I calculate the market shares of sellers at different price levels. First, I construct a price index as a measure of each seller's price level. For a seller indexed by s in market m and month t, the price index is equal to

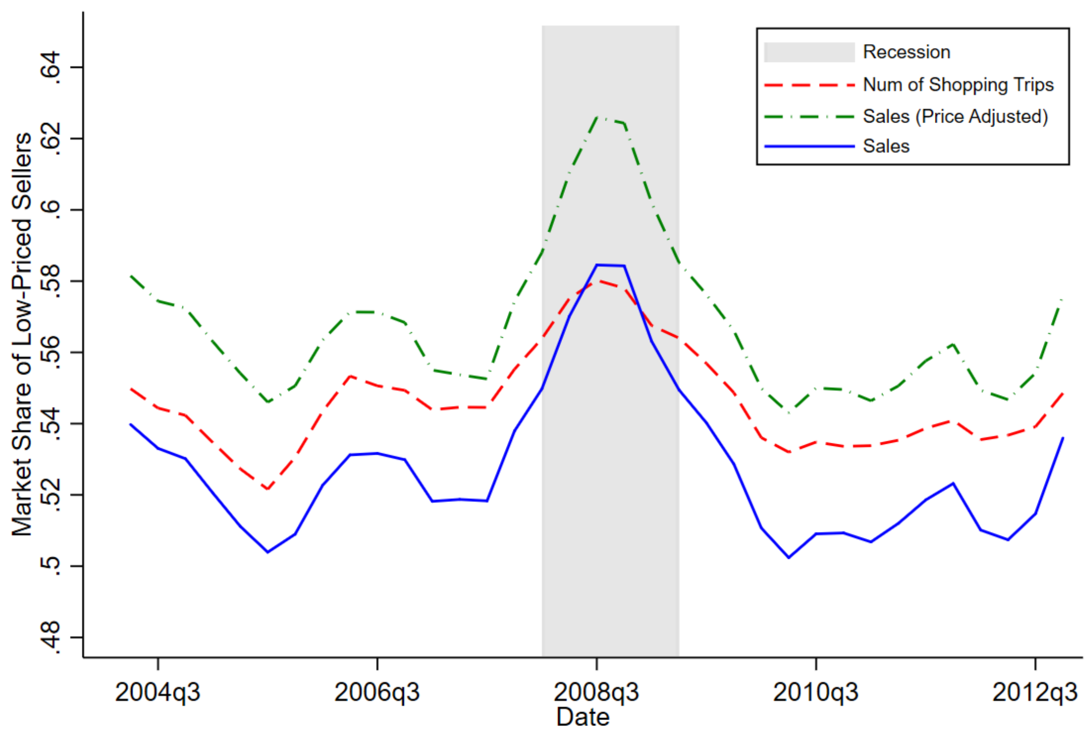
$$PI^s_{m,t} = \frac{\sum_i TS^s_{i,m,t}}{\sum_i \bar{p}_{i,m,t} q^s_{i,m,t}} \,,$$

where

$$\bar{p}_{i,m,t} = \frac{\sum_s TS^s_{i,m,t}}{\sum_s q^s_{i,m,t}} \,.$$

The nominator in this index is the total sales of the seller in a given period. The denominator is a counterfactual sales that if the seller sells all goods at the average price of the market. The price index measures the average expensiveness of the seller compared to all other sellers in the same market and period.

Figure 4: **Market Share of Low-Priced Sellers**



Notes: The figure plots the share of sales, price adjusted sales and shopping trips for the sellers in the bottom 50% of the price distribution. The share of shopping trips are computed using only stores presented in the consumer panel.

I identify ”low-priced sellers” in each month as the sellers whose price index is less than the median of price indexes of all sellers. I then calculate the share of sales of those low-priced sellers. As shown in the blue line of Figure 4, the market share of low-priced sellers increased from 52% to 58% during the Great Recession. As a robustness check, I calculate the price adjusted sales for each seller, which is equal to the denominator of the price index. The share of the price adjusted sales is shown by the green line, which increases significantly in the recession as well. Lastly, for the stores in the consumer panel, I also calculate the number of shopping trips. The red line shows the share of shopping trips for low-priced stores. The results in Figure 4 suggest that households switch to sellers with lower prices during the recession.

2.5 Customer Base and Prices

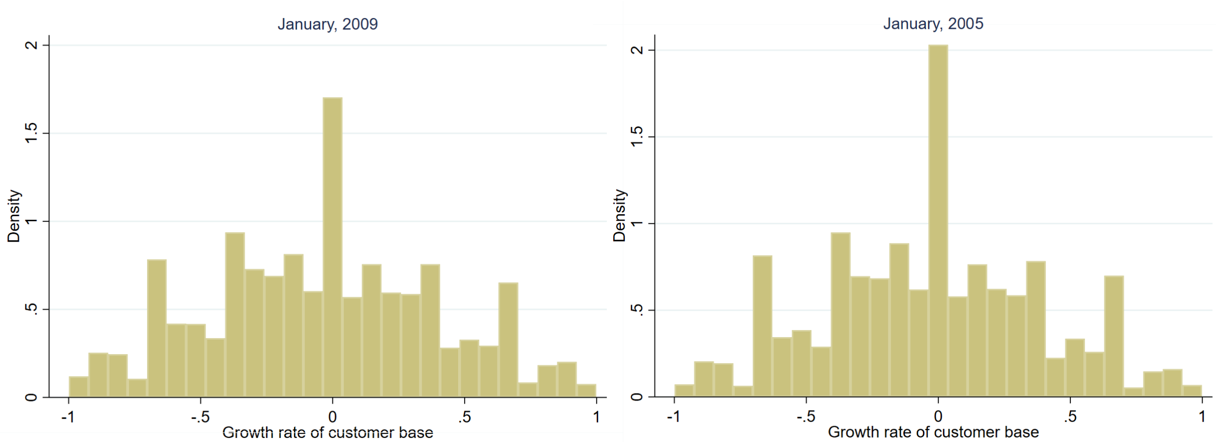
Households search and switch across sellers, and this will directly change the customer base of sellers. I relate the customer base growth rate to the change in posted prices in this section.

I measure the customer base of a given seller s in a month t as the number of shopping trips taken to the seller in the last three months, denoted as $NT_{s,m,t}$. An alternative measure used for robustness is the number of different households that visited the seller in the last three month, denote as $NH_{s,m,t}$. The growth rate of the customer base is then calculated as the log difference, which is written as

$$G^{NT}_{s,m,t} = log(NT^s_{m,t+1}) - log(NT^s_{m,t}) \,.$$

Figure 5 plots the pooled histogram of customer growth rates of all sellers in all markets, in January 2005. The distribution is symmetric but not standard. For comparison, the same histogram is shown for January 2009. For the month in the Great Recession, the mass is less concentrated around the middle.

Figure 5: **Distribution of Customer Base Growth Rates**



Notes: The figure plot the histogram of customer base rates for all sellers in January 2005 and in January 2009. Observations are not weighted.

I calculate the standard deviation of customer base growth rates over sellers in each geographical market and time period as a measure of dispersion. Each seller is weighted by their sales, such that

$$\left(\sigma^{NT}_{m,t}\right)^2 = \sum_s (G^{NT}_{s,m,t} - \bar{G}^{NT}_{s,m,t})^2 \frac{\sum_i TS^s_{i,m,t}}{\sum_{i,s} TS^s_{i,m,t}} \,.$$

During the Great Recession, the standard deviation of customer base growth rates increased by about 4%. This comes from a regression that regresses the log of standard deviation over a recession dummy and control for market fixed effects. As a robustness check, I also found similar results for the sales growth rate dispersion across sellers. The detailed regression are shown in Table A4 in the appendix.

Taking the price index constructed in section 2.4, I regress the change in the price index on the growth rate of the customer base to study if the customer base has an impact on price setting. I use the following specification:

$$\Delta PI^{s}_{m,t} = \alpha + \beta G^{NT}_{s,m,t} + D_s + D_{month} + \epsilon^{s}_{m,t} ,$$

where D_s is a seller dummy, D_m is a month dummy to adjust for seasonality. Intuitively, as household and sellers form long term relationship, an increase in customer base imply greater market power of the seller, so higher price will be posted. Hence, the coefficient of customer base growth rates is positive. However, in this regression, a simultaneous causality problem arises: sellers who post lower prices will attract more customers, so β could be downward biased. Since price change in a period does not impact the customer base growth in earlier periods, I regress price change on the one-period lag of customer base growth to eliminate the potential bias caused by simultaneous causality.

Table 3 reports the regression results for two measures of seller size separately. The coefficient of lagged customer base growth rate in column (3) is positive and significant, which suggests that sellers will increase their price posted after they have more customers. For robustness, I also regress the price change over sales change, which is a size measure of a seller that is related to the customer base. I check if the same relation holds between price and alternative size measures. The number in column (4) shows that sellers increase the price after an increase in sales in the previous period. More generally, the results in column (3) and (4) suggest that sellers raise their price after growing large. This price-setting pattern will convert an increased dispersion in customer base growth rate into a larger dispersion of price changes, which is key to the mechanism discussed in this paper.

Table 3: **Regression of Price Change on Customer Base Growth Rate**

	current period		1 period lag	
	(1)	(2)	(3)	(4)
$\Delta log(NT_{s,m,t})$	-0.0216**		0.0132**	
(shopping trips)	(0.0072)		(0.0039)	
$\Delta log(sales_{s,m,t})$		-0.0101***		0.0196***
(Sales)		(0.0007)		(0.0005)
Month FE	✓	✓	✓	✓
Seller FE	✓	✓	✓	✓
No. of Obs.	11,858	171,549	11,858	168,806
R^2	0.152	0.186	0.127	0.113

Notes: The table reports the regression results of price change of sellers on its customer base growth rates, controlling for geographical and time fixed effect. A seller is defined as a specific store in a market. I found similar results when define a seller defined at retailer level. Robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

3 Model

3.1 Model Environment

In this section, I build a general equilibrium model with consumer price search and firms accumulate customers. The economy consists of three types of agents: a representative producer, a continuum of firms of measure 1, called sellers, and a continuum of households of measure 1. Each household is attached to a seller, while a seller can have many households in its customer base. There are 2 types of goods in the economy: a final good for consumption, and an intermediate good. The consumption good is traded in a frictional goods market.

Two types of technologies characterize production. The representative firm produces intermediate goods linearly using labor, at aggregate productivity Z. Each seller has its idiosyncratic technology z, that it produces the consumption goods linearly from the intermediate goods. Each seller posts its price for the consumption good. The price of the intermediate good is normalized to 1.

The shopping process of each household takes 2 stages. In the first stage, each household observes the price posted by the attached seller. In the second stage, each household decides shopping effort, which is represented by probability $s \in [0, 1]$. With probability s, the household would search on the market, randomly find another seller, and observe its price p_2. The household then spends all the income on the cheaper seller for consumption goods. If a household purchases at the second seller, the household switches at the beginning of the next period. Otherwise, with probability $1 - s$, the household does not find another price and purchase at the current seller.

3.2 Household Problem

Households are assumed to be hand-to mouth. They choose shopping effort and labor to maximize the one period expected utility. Their preference in each period is given by

$$U(c_t, s_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \kappa \frac{s_t^{1+\phi}}{1+\phi} - \frac{l_t^{1+\psi}}{1+\psi},$$

where the first term is utility from consumption, and the rest two are dis-utility from shopping s and working l respectively.

Households are assumed to be ex-ante identical. Each household makes labor decision before it observes any prices of the consumption good, i.e., before shopping. In each period, a household first chooses labor to maximize the expected utility, taking price distribution F, wage w and profit from sellers π as given. Then, after observing the price posted by the affiliated seller, the household chooses optimal shopping effort. The shopping effort is represented by the probability of finding another seller during shopping. Lastly, the household spends all its income on consumption goods at the seller with the lowest price. The household problem can be stated as the household make labor supply decision and price contingent plans in shopping effort and consumption.

$$\max_{l, c(p_1, p_2), s(p_1)} E_{(p_1, p_2)} \left[U(c(p_1, p_2), s(p_1), l) \right] ,$$

s.t.,

$$c(p_1, p_2) \leq \begin{cases} \frac{wl+\pi}{p_1} & \text{with probability } s(p_1) \\ \frac{wl+\pi}{min\{p_1, p_2\}}, & \text{with probability } 1 - s(p_1) \end{cases} \quad \text{and } \; p_2 \sim F \, .$$

With probability s, the household does not find a second seller and thus purchase at current seller at price p_1. Otherwise, the household find another seller, observe price denoted as p_2, and purchase at seller who posts lower price.

3.3 Firms' Problem

There are 2 types of firms. A representative firm that produces intermediate goods, and heterogeneous sellers that produce and sell final goods. The representative firm hires workers, and produces intermediate goods using a linear technology in labor,

$$Y_{int} = Zl \, .$$

Wage is then equal to productivity,

$$w = Z \, .$$

The relevant state variables for the sellers' problem are its idiosyncratic productivity and size of the customer base. Sellers produce final consumption goods from intermediate goods using a linear technology. The idiosyncratic productivity z follows a Markov process:

$$log(z_t) = \rho log(z_t) + \epsilon_t, \; \epsilon_t \sim N(\mu, \sigma^2) \, .$$

I denote m as the customer base, which is defined as the mass of households who bought from the seller in the previous period. The price decision impacts sellers' value in two ways. First, it affects the level of profits per customer as in standard models. Second, the price affects the dynamics of the customer base. When a lower price is posted, less fraction of the customer base is lost, and more incoming customers are attracted. Moreover, the larger customer base is carried into future periods that allows the seller to make more profit. Let $\lambda(m, z)$ be the joint distribution of sellers over idiosyncratic productivity and customer base, the sellers' problem is as

$$W(m,z) = \max_p H(m,p)\tilde{\pi}(p) + \beta E[W(H(m,p),z')|z]\,,$$

where

$$\tilde{\pi}(p) = (p - \frac{1}{z})\frac{wl+\pi}{p}$$

and

$$H(m,p) = H_r(m,p) + H_n(m,p)\,.$$

$\tilde{\pi}(p)$ is the profit from a single customer. $H(m,p)$ is the total mass of households who bought from the seller in the current period. It is composed of two parts, the remaining households in the customer base, $H_r(m,p)$, and the new customers, $H_n(m,p)$. I discuss the detailed construction of $H(m,p)$ below.

Let $s(p)$ be the shopping policy function of the households. Then for households in the customer base, $s(p)$ fraction of them find a second seller. Given price distribution, $F(p)$ fraction of searching households find a lower price and leave. The share of remaining households is equal to $R(p) = (1 - s(p)F(p))$. The mass of remaining households is

$$H_r(m,p) = mR(p)\,.$$

The new customers are from the searching households whose first stage price is higher than the price posted by the seller. I refer these households as "valid households" in the rest part of the paper. Let $p(m,z)$ be the price policy function of all other sellers, then the total mass of valid households for the seller is

$$N(p) = \int \mathbf{1}_{\{p(\hat{m},z)>p\}} s(p(\hat{m},z))\hat{m}d\lambda(\hat{m},z)\,.$$

A certain fraction $h(m;\theta)$ of valid households discover the seller as they search. Then the mass of new customers is

$$H_n(m,p) = h(m;\theta)\int \mathbf{1}_{\{p(\hat{m},z)>p\}} s(p(\hat{m},z))\hat{m}d\lambda(\hat{m},z)\,.$$

In the customer base literate, the fraction of households that discover a given seller is assumed to be proportional to the size of customer base, which implies that $h(m;\theta)$ is linear in m. This assumption would make $H(m,p)$ a linear function in m, thus sellers' value function is homogeneous of degree 1 in customer base. As a result, the price function of sellers would not depend on the customer base. However, this is inconsistent with the fact in section 2.5, that sellers increase price posted after a growth in customer base. I express the probability that a given seller is observed in a random search as

$$h(m;\theta) = \frac{m^{\theta}}{\int \hat{m}^{\theta} d\lambda(\hat{m}, z)},$$

where I relax the assumption that this probability is linear in customer base m. The curvature of this probability over the customer base is decided by parameter θ, that I let data to discipline its value.

3.4 Equilibrium

In the model, each seller's price-setting rule is the optimal response to the price-setting rules of all other sellers. I consider a symmetric equilibrium, in which the price-setting rules are the same over sellers. The goods markets clear by construction, as each seller produces the consumption good according to the amount sold. Households solve for optimal shopping rule $s(p)$ taking price distribution as given. The shopping rule then enters each seller's optimization problem, in which each seller posts optimal price. Hence, price distribution in an equilibrium is consistent with sellers' price rule. I consider the case that the joint distribution over idiosyncratic productivity and customer base is stationary.

Definition 1. *A stationary equilibrium is consumer decision rule for shopping effort $s(p)$, seller decision rules $p(m,z)$ and $g(m,z)$, price distribution $F(p)$, seller distribution $\lambda(m,z)$, and fixed numbers (π, Z, w, l), such that,*

(1) Shopping decision rule $s(p)$ and labor l solve the household problem.

(2) $p(m,z)$ and $g(m,z)$ are solutions to the sellers' optimization problem.

(3) The sellers distribution is stationary and is consistent with sellers' decision rules:

$$\lambda(m, z) = \int \mathbf{1}_{\{g(\hat{m},\hat{z})\leq m\}} Prob(z' \leq z|\hat{z}) d\lambda(\hat{m}, \hat{z}) \, .$$

(4) Price distribution is consistent with sellers' decision rule for prices:

$$F(p) = \int \mathbf{1}_{\{p(m,z)\leq p\}} \frac{m^\theta}{\int \hat{m}^\theta d\lambda(\hat{m}, z)} d\lambda(m, z) \, .$$

(5) Profit distributed to households is consistent with profit made by sellers:

$$\pi = \int (p(m, z) - \frac{1}{z}) \frac{wl + \pi}{p(m, z)} g(m, z) d\lambda(m, z) \, .$$

The algorithm that solves the equilibrium numerically is described in appendix.

4 Analytical Results of the Model

Understanding how households make search decisions and how sellers set their price is crucial in understanding the mechanism that translates the change in shopping intensity to the increased dispersion of price growth rates. With further assumptions, several analytical results can be derived to illustrate the mechanism.

First, the decision of households on search intensity is characterized by the Euler equation of household problem.

Lemma 1 *Assume $\gamma > 1$ and $\phi > 0$. Then in any stationary equilibrium, shopping intensity $s(p_1; wl + \pi, F)$ satisfies the following condition, and is increasing in initial price and decreasing in income,*

$$s(p_1; wl + \pi, F) = \kappa^{-1/\phi} (wl + \pi)^{(1-\gamma)/\phi} \left(\int_{\underline{p}}^{p_1} F(p) p^{\gamma-2} dp \right)^{1/\phi} \, .$$

Proof of Lemma 1 is in the appendix. This lemma says that the shopping intensity is increasing in the price of the affiliated seller. It shapes the budget constraint of sellers. The law of motion of customer base $H(m, p)$ is decreasing in price, as household search more with a higher initial price.

Moreover, the Euler equation also implies that two forces drive the shopping intensity. On the one hand, there is an "income effect": if $\gamma > 1$, then shopping intensity is decreasing in income, consistent with the empirical facts documented in the literature.[13] On the other hand, there is a "dispersion effect": shopping intensity is increasing in price dispersion, as more dispersed price results in a higher return to shopping. It turns out that the "income effect" dominates the "dispersion effect" in my calibrated model. As a result, during a recession, as income drops, households search harder on average, thus are more likely to switch across sellers. Given households search for better prices, they switch to cheap sellers faster, hence increase the dispersion of customer base growth rate over sellers.

The rest of the transmission mechanism lies in the price-setting rule. The following proposition states that the price set by a seller is an increasing function of its customer base. This result implies that price change over sellers would become more dispersed when the change in customer base is more dispersed, which is key to the transmission of change in shopping behavior to change in price growth distribution.

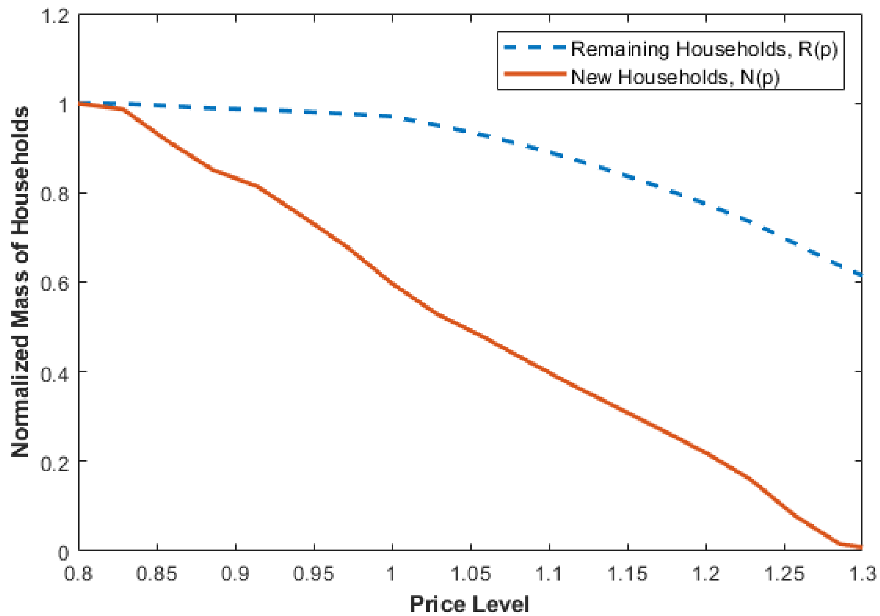
Proposition 1. *Suppose (1) $\theta \in (0, 1)$, (2) $F(p)$ is differentiable and (3) $-\frac{pH_{pp}(m,z)}{H_p(m,z)} < 2$. Then $\exists \underline{\kappa} > 0$ such that $\forall \kappa > \underline{\kappa}$, the price function $p(m, z)$ is increasing in m as a solution to sellers' problem.*

The proof of proposition 1 is in the appendix. Condition (2) and (3) rule out extreme cases that underlying price distribution is unusual, and thus demand elasticity of households change drastically within a small interval of price.

The intuition behind Proposition 1 is shown in Figure 6, which plots normalized mass of remaining households $R(p)$ and new customers $N(p)$ in the stationary equilibrium. For a given seller, suppose its customers switch very slowly, i.e., $s(p)$ is very small. Then a large fraction of households in its customer base does not observe another price and thus accept whatever the price posted. On the contrary, newly coming households already observe a price form another seller. The demand of households in customer base is then less elastic than the demand of new households. As shown in Figure 6, number of new customers is more responsive to a change in price. If $\theta < 1$, as a seller's customer base grows, a larger fraction of its current customer is from the customer base,

[13] See e.g. Kang (2018) for details.

Figure 6: **Mass of Two Types of Households as Functions of Price**



Notes: The figure plots the $R(p)$ and $N(p)$ in the stationary equilibrium, which are the mass of current households and new households respectively. Each function is normalized by dividing their value at the lower bound of the support.

so the seller faces less elastic demand and charges higher mark-ups. As θ gets smaller, this effect gets stronger because, for a seller, the composition of households is more sensitive to changes in size.

The demand elasticity a seller faces is also affected by the demand elasticity of each type of customer, which differs at different price levels. If this change of elasticity over price is moderate, then the composition effect discussed above is the main determinant of sellers' mark-ups. However, the intuition in Figure 6 may not work if the underlying price distribution is unusual. In an extreme case that the price distribution has a very sharp pike, then sellers around that pike will find the elasticity of demand change drastically as they shift price slightly. The condition (2) and (3) rule out this extreme case. Condition (2) says $F(p)$ is smooth without kinks and jumps. Condition (3) says that the law of motion of the customer base, which is shaped by the underlying price distribution, is not too convex.

5 Model Calibration

5.1 Computing Model Statistics

I estimate the model to match several aggregate moments in shopping behavior and cross-sectional facts on price and customer base. First, I show how I compute the model counterparts of data statistics.

Shopping intensity The model endogenously generates the shopping effort for new sellers of each household, $s(p_1; wl + \pi, F)$, which depend on income, price distribution and the first stage price. The average shopping effort for new sellers is then calculated as $\bar{s}(wl + \pi, F) = \int_{p_1} s(p_1; wl + \pi, F) dF(p)$, which implies that the fraction of new sellers is equal to

$$\frac{\bar{s}(wl + \pi, F)}{1 + \bar{s}(wl + \pi, F)} .$$

The total shopping effort in the model is defined as the number of sellers visited, which is equal to $1 + \bar{s}(wl + \pi, F)$. Substituting in the household Euler equation from the previous section, the elasticity of shopping effort over income is equal to

$$\begin{aligned} \epsilon^{income}_{shopping} &= \frac{\partial(1 + \bar{s}(wl + \pi, F))}{\partial(wl + \pi)} \frac{wl + \pi}{1 + \bar{s}(wl + \pi, F)} \\ &= \frac{1 - \gamma}{\phi} \frac{\bar{s}(wl + \pi, F)}{1 + \bar{s}(wl + \pi, F)} . \end{aligned}$$

This formula implies that $\frac{1-\gamma}{\phi}$ can be pinned down directly by the ratio between $\epsilon^{income}_{shopping}$ and the fraction of new sellers in the data. Lastly, the disperion of shopping effort, which is later used as one of the calibration target, is calculated as the coefficient of variation of $s(p_1, wl + \pi, F)$ over the price distribution.

Distribution of Price and Price Growth Rates The model solves the sellers' joint distribution over size and idiosyncratic productivity $\lambda(m, z)$, and the decision rule of price and customer base $p(m, z)$ and $g(m, z)$. The price distribution is then calculated directly using the formula in the condition (4) of equilibrium definition. The correlation between price and customer base is calculated as

$$Corr(p, m) = \frac{\int (p(m, z) - \bar{p})(m - \bar{m}) d\lambda(m, z)}{sd(p) sd(m)} .$$

I obtain price growth rates distribution through simulation. Given that sellers' distribution is consistent with the size decision rule $g(m, z)$ and the exogenous process of idiosyncratic productivity z, ergodic theory implies that sellers can be drawn by simulating one seller over many periods. I first draw a sequence of exogenous shocks $\{z_t\}_{t=1}^{T}$ for $T = 100,000$. Then I apply sellers' decision rules to obtain the sequences of customer base and prices, such that

$$p_t = p(m_t, z_t), \text{and } m_{t+1} = g(m_t, z_t) .$$

The growth rates are then computed as the log difference,

$$G_t^p = log(p_t) - log(p_{t-1}), \text{and } G_t^m = log(m_{t+1}) - log(m_t) .$$

I then regress G_t^p on G_{t-1}^m in order to obtain the model counterpart of the regression results shown in Table 3.

5.2 Parameters

I set a period in the model to be one quarter. Parts of the model parameters are set externally to fixed numbers, while the rest are calibrated to match data moments. Details are reported in Table 4. I set the discount factor $\beta = 0.99$ in line with the model period. The inverse of Frisch elasticity of labor supply is set to be $\psi = 0.5$, a value that is consistent with the micro and macro estimates in Keane and Rogerson (2012). According to the discussion in section 5.1, I set the risk aversion parameter γ so that the elasticity of shopping intensity to income is about 0.29, an estimate from Kang (2018) based on American Time Use Survey.

The remaining parameters are estimated to match several empirical moments from the consumer shopping behavior and cross-sectional distribution of price and customer base. While the parameters are estimated jointly, the most closely related moment for each parameter is reported in Table 4.

Table 4: **Model Parameter Values**

Parameters	Description	Value	Target/Source
Set Externally			
μ	Mean of seller productivity shock	0	
γ	Risk aversion coefficient	3	Kang (2018)
β	Discount factor	0.99	Model period=1 quarter
ψ	Inverse of elasticity of labor supply	2	Keane and Rogerson (2012)
Calibrated			
ρ	Persistence of seller productivity shock	0.89	Corr(price, customer base)=-0.281
σ	SD of seller productivity shock	0.18	SD of prices of identical goods=0.187
κ	Disutility of shopping	0.87	Fraction of new sellers=14.6%
ϕ	Curvature of shopping disutility	1.05	CV(new sellers)=1.397
θ	Curvature of search probability over customer base	0.38	Coef. of customer base growth over price change (Table 3)

The sellers' idiosyncratic productivity shocks are the source of seller heterogeneity in this model. The joint distribution of sellers over productivity and customer base is then translated to the distribution of price by their pricing rule. Hence, I calibrate the standard deviation of seller productivity shock, σ, to match the cross-sectional dispersion of prices for identical goods (UPCs) in the data. The detailed calculation of this empirical moment is shown in the appendix. The persistence parameter ρ affects the correlation between productivity and customer base in the stationary distribution. If the idiosyncratic shocks are very persistent, then a seller is likely to maintain a high productivity level for a long time, during which it could accumulate customers by charging a lower price. Since price is a function of customer base and sellers' productivity in the model, the persistence parameter will impact the correlation between price and customer base in the stationary equilibrium.

The set of shopping parameters is calibrated to match several empirical moments. As in section 2.4, the fraction of new sellers is 14.6%. I set κ such that the model produces the same ratio in the stationary equilibrium. The sensitivity of household shopping intensity to first stage price p_1 is determined by the curvature of dis-utility of shopping, ϕ. From the household Euler equation, the curvature parameter affects the dispersion of shopping intensity over households for a given price distribution. I then calibrate this parameter to match the cross-sectional coefficient of variation of the share of new sellers across households.

Lastly, as discussed in section 3.3, the curvature parameter of the observing probability function for sellers, θ, is set to match the coefficient of customer base growth over price change shown in Table 3.

5.3 Model Fit

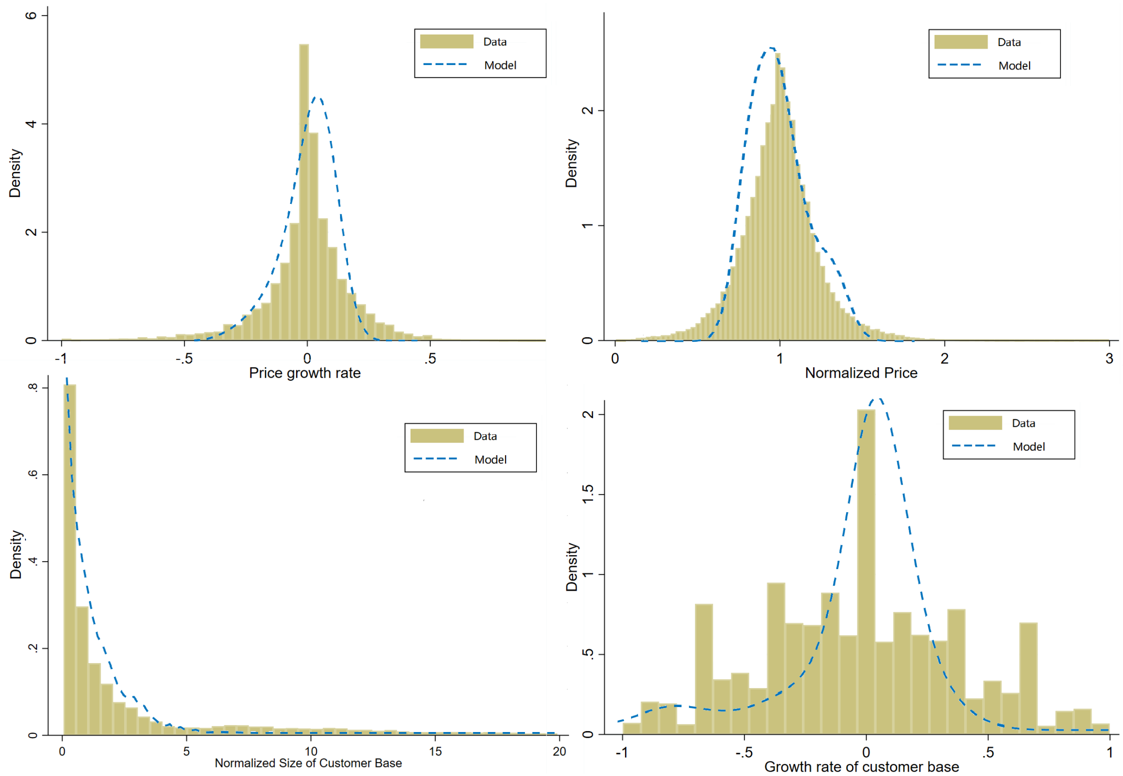
Table 5 summaries the statistics generated by the model and their data counterparts. The model is calibrated by targeting five moments discussed in the section above, and the model matches these moments well. Several relevant moments, such as the dispersion of price growth rates, are not targeted. The table also shows the model fitness to those non-targeted moments. The standard deviation and interquartile range of price growth rate in the model are both close to data. The model also has a good fit for the second moment of both the customer base and its growth rate.

Table 5: **Model Fit**

Moments/Statistics	Related Parameters	Data	Model
Targeted			
Corr(price, customer base)	ρ	-0.281	-0.297
Sd of prices of identical goods	σ	0.187	0.188
Fraction of new sellers	κ	14.6%	14.1%
CV(new sellers)	ϕ	1.397	1.226
Coef. of customer base growth over price change (Table 3)	θ	0.0132	0.0130
Non-Targeted			
Sd of price growth rate		0.129	0.098
IQR of price growth rate		0.070	0.062
CV of customer base		1.091	1.026
Sd of customer base growth rate		0.212	0.260

Notes: The standard deviations, IQRs and coefficient of variations from data are the average numbers over all months in the year 2005.

Figure 7: **Model vs Data: Price Distribution and Size Distribution**



Notes: The figure compares price growth distribution (top-left), price distribution (top-right), size distribution (bottom-left) and size growth distribution (bottom-right) from data with model outcomes. The prices and price growth rates are from January 2005. Histograms are from pooled data for all goods in all markets. In the price distribution plot, price observations for each good and market is normalized by dividing to the average price of the given good in the given market.

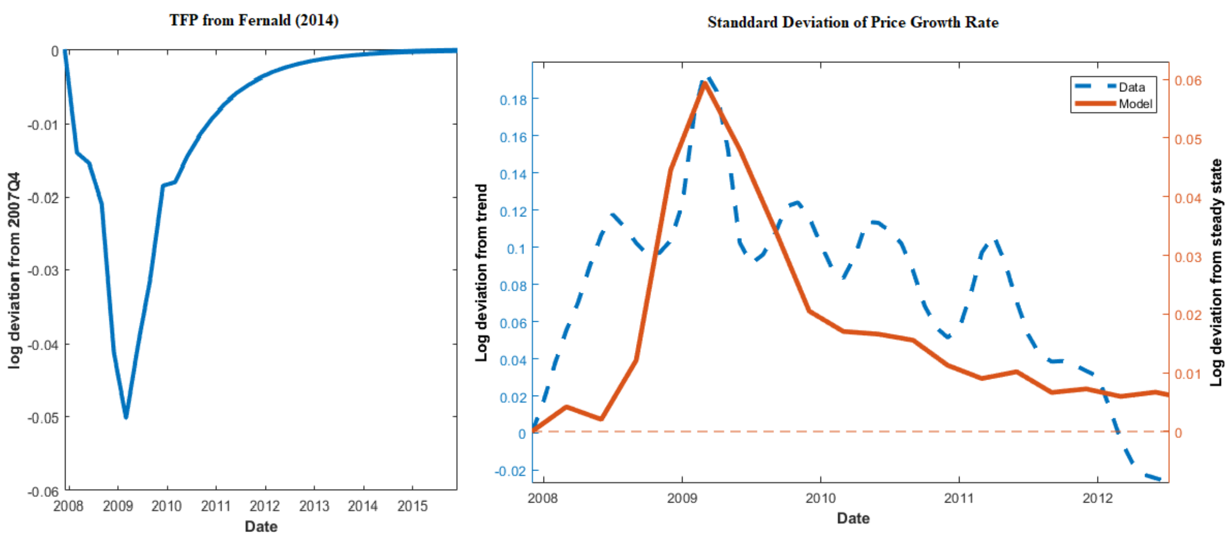
Figure 7 shows the model fitting to the entire distributions of price and the customer base. I plot the empirical distributions as the histograms of pooled data in the year of 2005. The model generates a right-skewed size distribution of sellers that matches its empirical counterpart (bottom-left) well. Lastly, the model produces price growth distribution (top-left) and price distribution (top-right) close to the ones in the data.

6 Quantitative Results

6.1 Counter-cyclical dispersion of price growth rate

The primary focus of this paper is to quantify the impact of shopping behavior on the price growth distribution. The main quantitative exercise in the paper is to study the transition path of the economy after unexpected shocks of deterministic sequence to aggregate productivity, $\{Z_t\}_{t=1}^{T}$. I set productivity of the first 10 quarters using empirical TFP measure starting from the first quarter of 2008. The empirical TFP measure is from the estimates in Fernald (2014). The rest of the process is generated by the deterministic first-order autoregressive process $log(Z_t) = \rho_z log(Z_{t-1})$ with $\rho_z = 0.9$.

Figure 8: **Transition Path of Price Growth Dispersion vs Data**



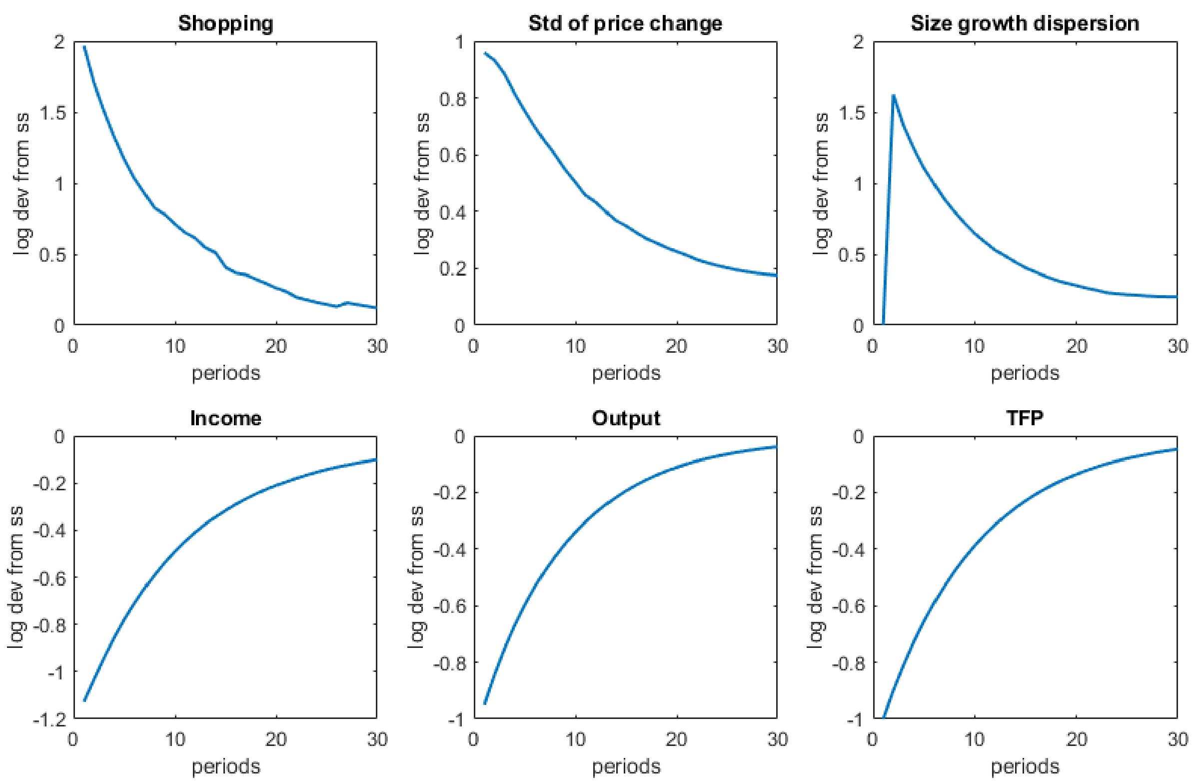
Notes: The first 10 periods of the left figure is based on the estimated TFP process in Fernald (2014). The numbers are calculated in the form of log deviation from 2007Q4 after removing the growth of trend. In the rest of periods, the process is generated by a deterministic first-order autoregressive process $log(Z_t) = \rho_z log(Z_{t-1})$ with $\rho_z = 0.9$.

Figure 8 shows the standard deviation of price growth distribution in the Great Recession experiment. For comparison, the blue dash line is log deviation from trend in the data, while the red line is the model outcome in log deviation from steady-state. The model can produce a sharp increase in price growth dispersion during the Great Recession as in the data. The standard deviation of the price growth rate increases by about 18.5% during the Great Recession. The corresponding increase produced by the model is around 6.1%, which is about 30% of the size in the data.

6.2 Illustration of Model Mechanism

To illustrate the model mechanism, I calculate the transitional path when setting first-period productivity to -1%, an initial one percent drop in the productivity of the representative firm. In the rest periods, aggregate productivity is generated by the same deterministic first-order autoregressive process as in the previous exercise.

Figure 9: **Transition Path after -1% Productivity Shock**



Notes: The transition path are computed by setting the the productivity of representative firms equal to 0.99 in the first period. In the rest of periods, the productivity process is generated by a deterministic first-order autoregressive process $log(Z_t) = \rho_z log(Z_{t-1})$ with $\rho_z = 0.9$.

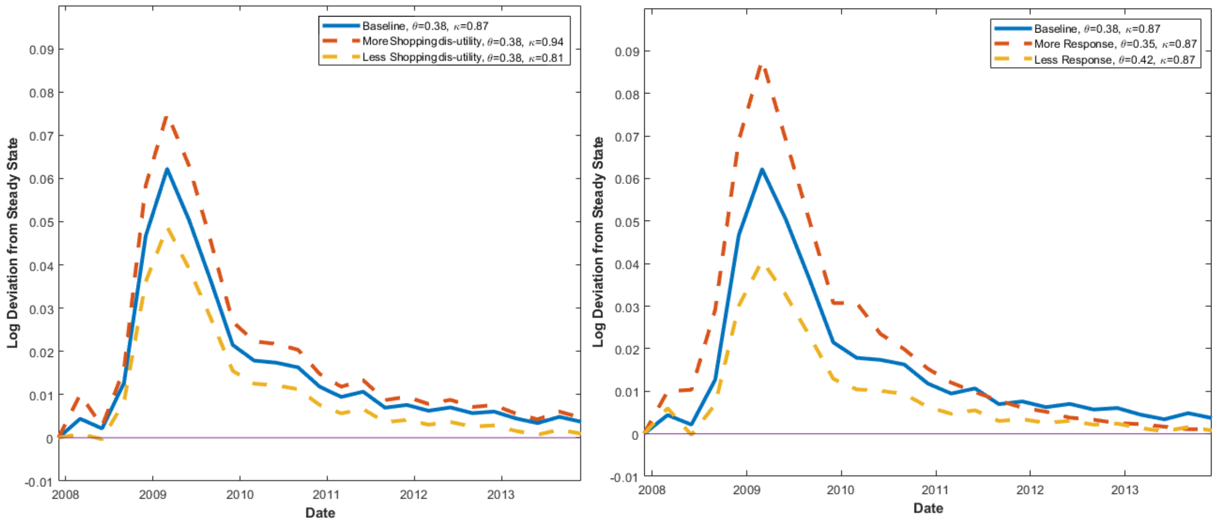
Figure 9 shows the transition path of several variables after a 1 percent negative productivity shock. The sudden drop in aggregate productivity reduces wage, as well as household income. Households reduce consumption, and search more as the higher marginal utility of consumption makes a lower price more appealing. As shown in the top left block of the figure, average shopping intensity increased by 2%. Compared to stationary equilibrium, cheap sellers, which are mostly large and productive in the model, get a larger inflow of customers, while expensive sellers lose more customers. The price response of a seller depends on changes in its customer base and idiosyncratic productivity. While the latter is exogenous and follows the same process as in stationary equilibrium, the distribution of customer base change is more dispersed. Since sellers' price function is increasing in the customer base, the increased dispersion of customer base change results in the more dispersed price growth rates across sellers.

6.3 Robustness Check

The sensitivity of sellers' price decision to change in the customer base is the key element that decides the strength of the mechanism. As discussed in Proposition 1, the parameters related are dis-utility of shopping, κ, and the curvature of search probability over customer base, θ. First, larger dis-utility of search makes it harder to switch. Hence, market power is affected more by the size of the customer base. Second, with a lower θ, the fraction of captive households would increase more after a growth in the customer base, so there would be a larger increase in the market power of sellers. In both cases, the price of a seller is more sensitive to the size of its customer base.

As a robustness check, I redo the Great Recession experiment using different values of κ and θ. First, as κ is set to match the fraction of new sellers calculated in section 2, I set $(\underline{\kappa}, \bar{\kappa}) = (0.81, 0, 94)$, so that the model respectively matches the lower and upper bounds of the one standard deviation interval of the empirical moment. As shown in the left part of Figure 10, the rise in price growth dispersion reduces from 6.1% to 4.9% after setting a smaller $\kappa = \underline{\kappa}$. Similarly, a larger $\kappa = \bar{\kappa}$ results in a larger increase in price growth rate dispersion. Second, I use the same method to vary θ, and the results are shown in the right part in Figure 10. The model produces a larger increase in the dispersion of price growth rates compared to baseline when θ is lower, which is consistent with the intuition discussed in Proposition 1. By varying the key parameters, the model generates an increased price growth dispersion between 4.3% to 8.8%, which are between 23% to 48% of the size in the data.

Figure 10: **Transition Path of Price Growth Dispersion**



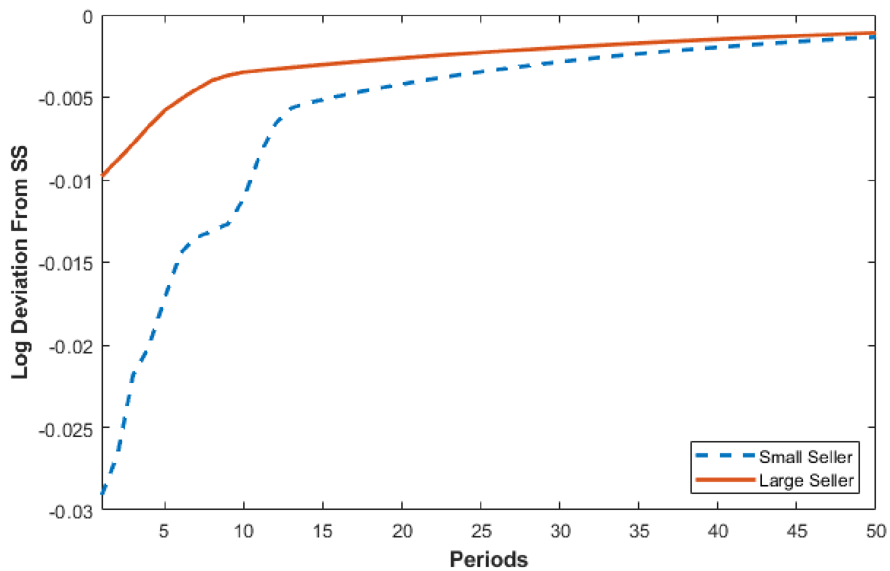
Notes: The transition path are computed by solving the stationary equilibrium with alternative parameter values, and the compute transition path for the same TFP sequence as estimated in Fernald (2014).

6.4 Implications of Firm Dynamics

The model has implications on the price dynamics of sellers of different sizes. Small sellers are more responsive to aggregate productivity shocks compared to large sellers in the model because the search intensity of their affiliated households is higher. During the recessions, both large and small sellers reduce price because higher search intensity of households intensifies price competition. Sellers with higher price suffer more from intensified competition since their affiliated households are more likely to leave upon finding another seller. Given that large sellers are on average more productive and set lower prices in the model, large sellers reduce price less compared to small sellers.

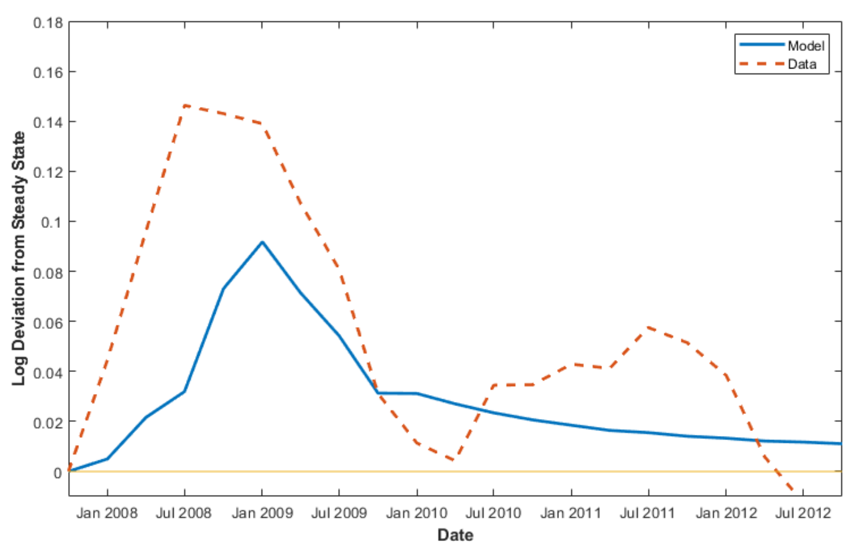
Figure 11 shows the price dynamics for sellers at the 25th and 75th percentile of the size distribution after a negative 1% shock in the productivity of the representative firm. While they set lower prices upon a negative TFP shock, the small seller reduces price more compared to the large seller. Similarly, after a positive productivity shock, both types of sellers benefit from the decreased search effort of households. Small sellers benefit more for the same reason and, as a result, increase the price more.

Figure 11: **Price Response of Large and Small Sellers**



Notes: The initial size of the two sellers are equal to the 25th and 75th percentile of size distribution in the stationary equilibrium. The initial productivity of each seller is equal to the average productivity of sellers of the same size in the stationary equilibrium. The aggregate shock is 1% negative TFP process described in section 6. The idiosyncratic productivity of each seller is assumed to be equal to its initial values over periods.

Figure 12: **Sales Growth Dispersion: Model vs Data**



Notes: The blue line the model outcome of standard deviation of sales growth dispersion in the Great Recession experiment. The red dashed line is the data measure of standard deviation of sales growth rates across sellers. For comparison, data is calculated in log deviation from trend, and the model outcome is in the form log deviation from steady state.

The model also implies counter-cyclical sales growth dispersion, which is consistent with the empirical findings in the literature. The model generates counter-cyclical customer base growth dispersion because consumers are switching faster to sellers that post lower prices. Since the sales of a seller are proportional to its customer base in the model, the sales growth dispersion also rises during recessions. As shown in Figure 12, the empirical standard deviation of the sales growth rate in the retailer scanner data rises about 15% during the Great Recession. Comparatively, the model generates a 10% increase in the standard deviation of sales growth rates in the quantitative exercise discussed in section 6.1, which explains about 60% of the rise in the data.

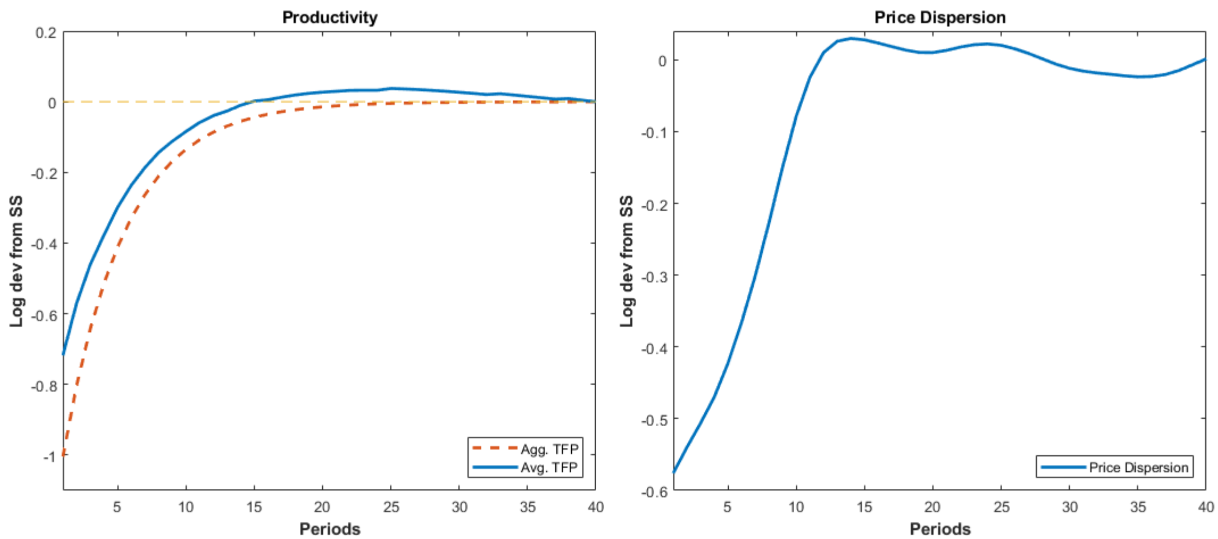
6.5 Reallocation and Welfare

The model shows that increased dispersion can arise from the heterogeneous response of sellers to the same TFP shock. The welfare implication for this is very different from the case in which the increased dispersion is a result of uncertainty shocks. In the latter case, greater heterogeneity at the firm level increases price dispersion, and results in greater misallocation. This is consistent with the discussion in Woodford (2003), that larger price dispersion implies greater welfare loss in the business cycle.

In comparison, the mechanism in this paper reduces welfare cost of the business cycle by reallocating production among sellers with different productivity. In the model, productive sellers post lower prices in order to attract more consumers. During a recession, as households switch to relatively cheap sellers, production is reallocated to more productive sellers. This intermediately increases efficiency, and the average productivity will not drop as much as the aggregate TFP. Moreover, because households are attached to sellers, households who switch to a more productive seller will not turn back after the recession. As the aggregate TFP goes back to its steady-state level, the average productively overshoots. The more persistent the idiosyncratic productivity is, the longer it takes for the average productively back to its level in the stationary equilibrium. Hence, the welfare gain lasts even after the recession.

Figure 13 illustrates the reallocation of production by showing the response of average productivity after a negative 1% drop in aggregate TFP. Given linear technology, I calculate the average

Figure 13: **Average Productivity and Price Dispersion**



Notes: The figure shows the response of average productivity (left) and price dispersion (right) after a negative 1% shock in aggregate TFP. The average productivity is calculated as the total consumption goods over labor.

productivity in the model as the total final goods produced over labor. As an immediate response, the decline in average productivity is about 70% of the drop in aggregate TFP. In the long-term, the efficiency gain from production reallocation vanishes slowly, so the average productively overshoots.

7 Conclusion

This paper documents several new facts on shopping behavior and price dynamics. First, households form a repeated-purchase relationship with sellers, and they switch over sellers faster during recessions. Second, all else equal, sellers set higher prices as their customer base grows. Motivated by these facts, I propose a novel mechanism that links price volatility to consumer shopping behavior. During recessions, consumers search harder for prices, thus switch over sellers more frequently. This raises the dispersion of the size growth rate across sellers. Since each seller increases the price after a growth in its customer base, the increased dispersion in size would result in a more dispersed price change.

I build a general equilibrium model with household endogenously make shopping decisions and firms accumulating customers. The calibrated model shows that the mechanism is quantitatively important in explaining the dynamics of price volatility, and reduces welfare cost of business cycle by reallocating production to more productive firms. The model also provides explanations for two related empirical facts in the literature: (1) counter-cyclical sales growth dispersion, and (2) small firms are more responsive to aggregate shocks than large firms.

Besides price change, literature also documents counter-cyclical behavior of economic variables such as sales growth, unemployment growth, and revenue-measured TFP. This paper links sales growth to price change by studying sellers' price-setting behavior. It is worth exploring how the counter-cyclical dispersion of price growth, unemployment growth, and TFP may be related, which could be important in understanding the mechanisms in the business cycle. Moreover, I analyze the price dynamics in a model without price rigidity. It would be interesting to study the implications of consumer search on the aggregate dynamics when the price is sticky. I leave these for future research.

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Appendix

A Empirical Analysis

A.1 Data summary

Table A1: **Summary Statistics of Data**

		numbers per market, monthly average				
Year	# of markets	Stores	Observations	Goods (UPC)	Households	Trips
2001	50	40.9	94.2	8146.3	3149.3	29.9
2002	50	43.3	91.6	8217.9	3365.1	34.5
2003	50	40.9	95.4	8657.4	3343.7	33.6
2004	50	40.5	96.7	9152.3	3198.1	30.7
2005	50	41.9	95.6	9096.2	3303.6	31.2
2006	50	41.1	98.4	9199.7	3345.1	31.1
2007	50	42.5	104.2	11204.6	3254.7	29.1
2008	50	41.2	113.3	11445.4	3415.0	31.8
2009	50	41.2	114.7	11543.7	3379.3	30.1
2010	50	40.4	113.7	11249.2	3354.7	29.9
2011	50	38.4	114.1	11348.9	3250.1	28.6
2012	50	39.6	108.3	11293.5	2734.1	24.2

Notes: This table shows the average numbers of stores, transactions, goods, households and shopping trips in a market. The numbers are calculated for each month and then averaged over months in a year. The middle 3 columns are for scanner data and right two columns summarize consumer panel. Numbers of trips and observations are in the unit of thousands. The numbers are calculated after imposing sample selection criteria in section 2.2.

A.2 Price Growth Dispersion

Table A2: **Dynamics of Price Growth Rate Dispersion**

	Dispersion measures			
	(1) Std. weighted	(2) IQR weighted	(3) Std. non-weighted	(4) IQR non-weighted
$D^{recession}$	0.0155**	0.0153***	0.0458***	0.0346**
	(0.0070)	(0.0017)	(0.0080)	(0.0132)
Month FE	✓	✓	✓	✓
Geographical FE	✓	✓	✓	✓
Num of Obs.	6,600	6,600	6,600	6,600
R^2	0.2450	0.4416	0.1650	0.3035
Mean Dispersion in Non-Recession Periods	0.1269	0.0688	0.1849	0.0903
	(0.0274)	(0.0495)	(0.0306)	(0.0681)
Change	+12.2%	+22.3%	+27.8%	+38.4%

Notes: The table reports the regression results of price growth dispersion measures over a recession dummy, after control for geographical variation and seasonal effect. The table also report the mean and standard deviation of the two dispersion measure during non-recession periods. The last line of table shows the percentage change in the recession compared to non-recession periods. Robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Using calculated standard deviation and inter-quantile range of price growth rates, I estimate the following equation in OLS,

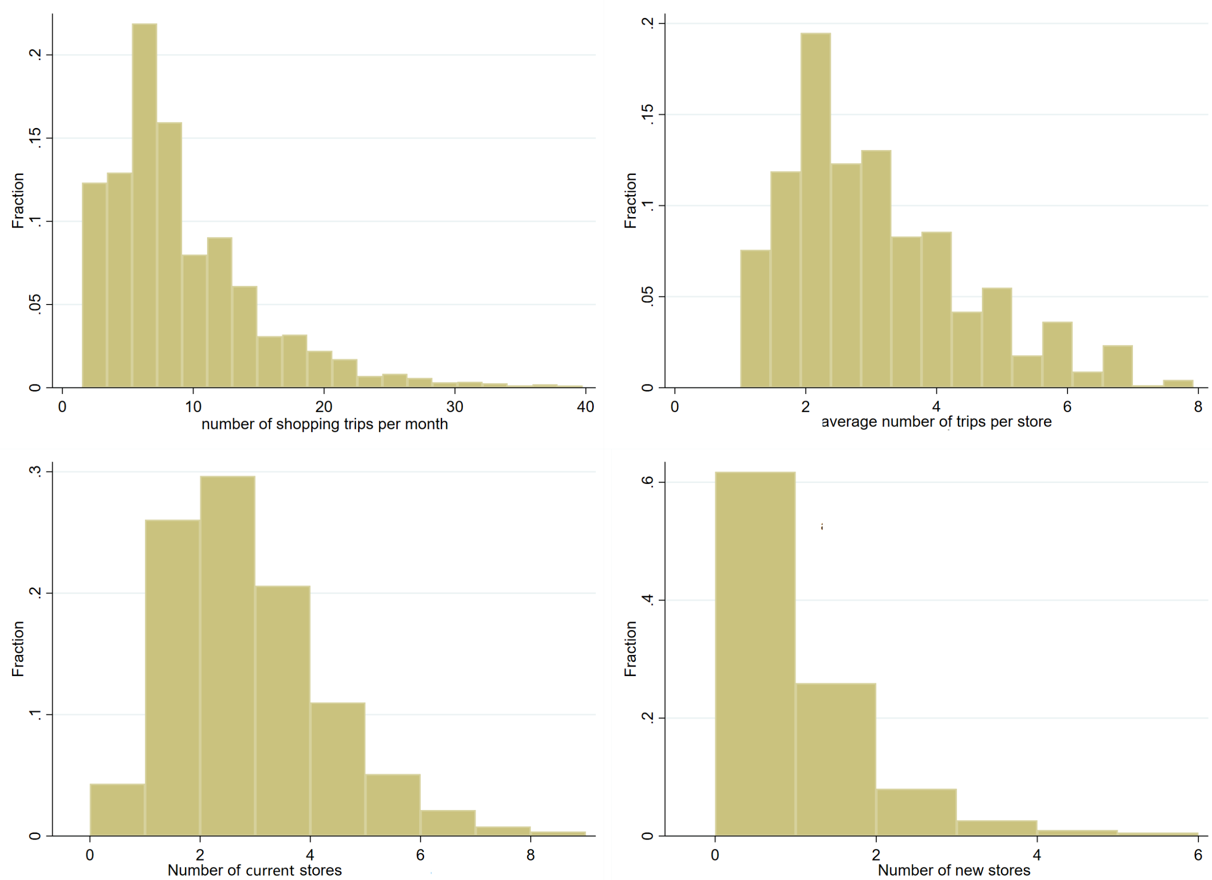
$$\sigma_{m,t} = \alpha + \beta D_t^{recession} + \gamma_1 D_m + \gamma_2 S_t + \epsilon_{m,t}$$

Where m indexes the geographical market. $D^{recession}$ and D_m are dummy variables for recession and market respectively. M_t is a month dummy used to control for seasonality. Table A2 reports the coefficient of recession dummy and average dispersion in non-recession periods. The geographical averages of standard deviations and IQRs over time is plotted in Figure 1.

A.3 Shopping Intensity

Table A3 shows the summary statistics for household shopping behavior using different sample criteria. A seller is defined as a store in part (1) and as a retailer in part (2). Figure A1 shows the pooled distribution of each of the shopping intensity margin for all households in all years.

Figure A1: **Shopping Intensity in 3 Margins**



Notes: This Figure plots the histograms of number of shopping trips per month (top left), number of shopping trip per store (intensive margin, top right), number of different stores that was visited previously (persistent extensive margin, bottom left), and number of different new stores visited (temporary extensive margin, bottom right). The numbers are calculated for each household in each month. I use the threshold of purchase/income ratio equal to 10% in this figure.

Table A3: **Summary Statistics of Shopping Intensity Margins**

(1) Decomposition at store level

purchase/income threshold	monthly average number of: shopping trips	shopping trips per seller	different sellers	current sellers	new sellers
0%	9.23	3.57	2.94	2.51	0.43
	(7.46)	(2.56)	(1.76)	(1.58)	(0.75)
10%	9.98	3.60	3.02	2.58	0.44
	(7.79)	(2.63)	(1.83)	(1.60)	(0.76)
20%	11.17	3.67	3.14	2.69	0.45
	(8.53)	(2.71)	(1.95)	(1.77)	(0.76)
30%	11.93	3.73	3.19	2.74	0.45
	(8.96)	(2.74)	(1.94)	(1.75)	(0.77)

(2) Decomposition at retailer level

purchase/income threshold	monthly average number of: shopping trips	shopping trips per seller	different sellers	current sellers	new sellers
0%	9.23	3.57	2.95	2.52	0.43
	(7.46)	(2.56)	(1.76)	(1.58)	(0.75)
10%	9.98	3.61	3.03	2.59	0.44
	(7.79)	(2.63)	(1.82)	(1.61)	(0.76)
20%	11.17	3.67	3.14	2.69	0.45
	(8.53)	(2.71)	(1.94)	(1.77)	(0.76)
30%	11.93	3.74	3.20	2.75	0.45
	(8.96)	(2.73)	(1.94)	(1.76)	(0.77)

Notes: Numbers are calculated by averaging over households in a month and then averaging over months in year 2011. Numbers in the brackets are standard deviations across households. Current sellers are identified as sellers visited in the last 3 month. Sellers are defined as (1) stores (2) retailers in a market.

A.4 Customer Base Growth Rates over Time

I estimate the following regression,

$$log(\sigma_{m,t}^{NT}) = \alpha + \beta D_{recession} + \gamma_1 D_m + \gamma_2 D_{month} + \epsilon_t^s$$

Where $\sigma^{NT}_{m,t}$ is the standard deviation customer base of growth rate. $D_{recession}$ and D_m are dummy variables for recession and market respectively. As there are only 2 markets in consumer panel, I use zip codes of visiting households to identify store location, so that there are more geographical variations for identification in the time series regression. D_{month} are month dummies that adjust for seasonality. Table A4 reports the coefficient of recession dummy.

Table A4: **Dispersion of customer Base Growth Rate in Recession**

	customer base/size measures		
	(1) number of shopping trips $\sigma^{NT}_{s,m,t}$	(2) number of households $\sigma^{NH}_{s,m,t}$	(3) monthly total sales $\sigma^{sales}_{s,m,t}$
$D^{recession}$	0.0854*** (0.0127)	0.1009** (0.0373)	0.0788* (0.0434)
Month FE	✓	✓	✓
Geographical FE	✓	✓	✓
Num of Obs.	1,056	1,056	5,924
R^2	0.6183	0.5119	0.1828

Notes: The table report the regression results of customer base growth dispersion and sales growth disperion on a recession dummy after control for geographical variation. A seller is defined as a specific stores. I found similar results for sales growth dispersion when define a seller as stores under same supply chain in the same market. Robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

A.5 Price Dispersion of Identical Goods

Taking the prices calculated from scanner data in section 2.3, I define the normalized price as the ratio between unit price to the average price. As stores are of different sizes, I use $W^s_{i,m,t}$ to represent the weight I assign to different stores. In my baseline, I use uniform weights.

$$\tilde{p}^s_{i,m,t} = \frac{p^s_{i,m,t}}{\bar{p}^s_{i,m,t}}.$$

I calculate statistics of distributions for each good and then weighted across goods using sales. For each given good i, market m, and month t, I calculate the standard deviation of the $\tilde{p}^s_{i,m,t}$ as

$$\mu^s_{i,m,t} = \sum_s \tilde{p}^s_{i,m,t} W^s_{i,m,t} \quad \sigma^s_{i,m,t} = \left[\sum_s W^s_{i,m,t} (\tilde{p}^s_{i,m,t} - \mu^s_{i,m,t})^2 \right]^{1/2}$$

I take averages of the standard deviation over goods using share of sales as weights. I calculate the share of sales for good i, market m, and month t as

$$S_{i,m,t} = \frac{\sum_s TS^s_{i,m,t}}{\sum_i \sum_s TS^s_{i,m,t}} \,.$$

Then the weighted standard deviations are calculated as,

$$\sigma^s_{m,t} = \sum_s \sigma^s_{i,m,t} S_{i,m,t} \,.$$

B Algorithms

B.1 Solving for Stationary Equilibrium

Solving the stationary equilibrium involves finding the price rule $p(m, z)$ that satisfies the optimization condition of all sellers. I iterate by guessing a price rule $p(m, z)$ for sellers, and then solve the sellers' problem which gives an updated price rule. However, sellers' budget constraint depend on not only the price rule of all other sellers, but also the household shopping policy function $s(p)$ and the stationary distribution $\lambda(m, z)$. For each guess of price rule $p(m, z)$, I construct sellers budget constraint by figuring out the policy function of customer base $g(m, z)$, shopping intensity $s(p)$ and stationary distribution $\lambda(m, z)$ that is consistent with household problem. Computational algorithm is stated below.

Algorithm:

1. Guess a price rule for sellers $p(m, z)$.

2. Solve $g(m, z)$, $s(p)$ and $\lambda(m, z)$ that is consistent with guessed price rule $p(m, z)$ and household problem.

   2.1 Guess a policy function of customer base $g(m, z)$, calculate stationary distribution using equilibrium condition (3). Then calculate posted price distribution using equilibrium condition (4).

   2.2 Solve profit π as a function of l using equilibrium condition (5). Solve household problem, obtain shopping rules $s(p)$.

   2.3 Recover the new policy function of customer base from seller's budget constraint, compare with the guess in step 2.1, update the guess.

3. Solve sellers problem, obtain new price rule, $p^{new}(m, z)$. Stop if $||p^{new}(m, z) - p^{new}(m, z)|| < 0.001$. Otherwise, update the guess according to the equation below. I use $\alpha = 0.3$.

$$p(m, z) = (1 - \alpha)p(m, z) + \alpha p^{new}(m, z)$$

B.2 Solving for a Transitional Path

The following steps outline the algorithm used.

Algorithm:

1. Fix the transition period T, guess a price rule for sellers in each period $\{p_t(m,z)\}_{t=1}^T$.

2. *Forward iteration.* For every period, Solve $g_t(m,z)$, $s_t(p)$ and $\lambda_t(m,z)$ that is consistent with guessed price rule $p_t(m,z)$, household problem and last period seller distribution $\lambda_{t-1}(m,z)$.

   2.1 Guess a policy function of customer base $g_t(m,z)$, calculate $\lambda_t(m,z)$ using $\lambda_{t-1}(m,z)$. Calculate posted price distribution from $g_t(m,z), p_t(m,z)$ and $\lambda_t(m,z)$.

   2.2 Solve profit π_t as a function of l_t using equilibrium condition (5). Solve household problem, obtain shopping rules $s_t(p)$.

   2.3 Recover the new policy function of customer base $g_t^{new}(m,z)$ from seller's budget constraint, compare with the guess in step 2.1, update the guess.

3. *Backward iteration.* Solve sellers problem, with sellers' value function $W_T(m,z)$ equal to the value function from stationary equilibrium, and each period budget constraint calculated from $\{g_t(m,z), p_t(m,z), s_t(p), \lambda_t(m,z)\}_{t=1}^T$. Obtain new price rule, $\{p_t^{new}(m,z)\}_{t=1}^T$. Stop if $||p_t^{new}(m,z) - p_t^{new}(m,z)|| < 0.001, \ \forall t$. Otherwise, update the guess according to the equation below.

$$p_t(m,z) = (1-\alpha)p_t(m,z) + \alpha p_t^{new}(m,z)$$

C Additional Model Outcomes

Figure A2: **Stationary distribution of Productively and Customer Base**

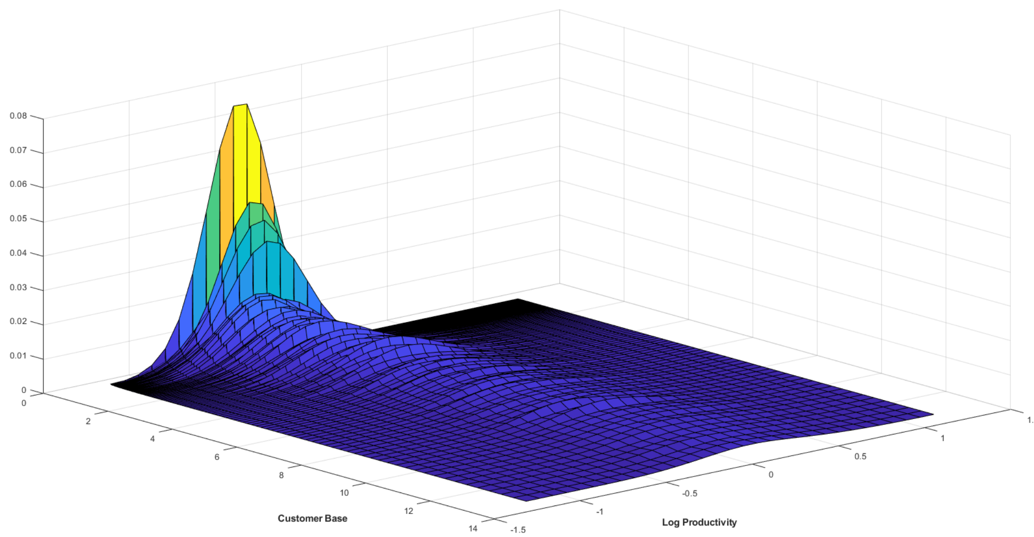


Figure A3: **Stationary Price Decision Rule**

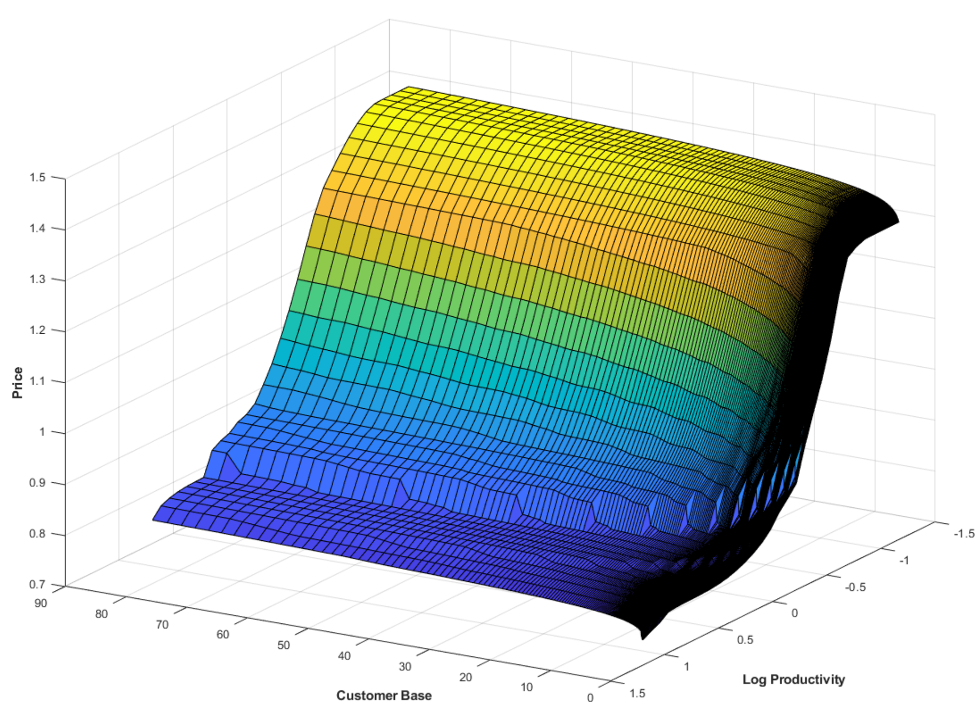
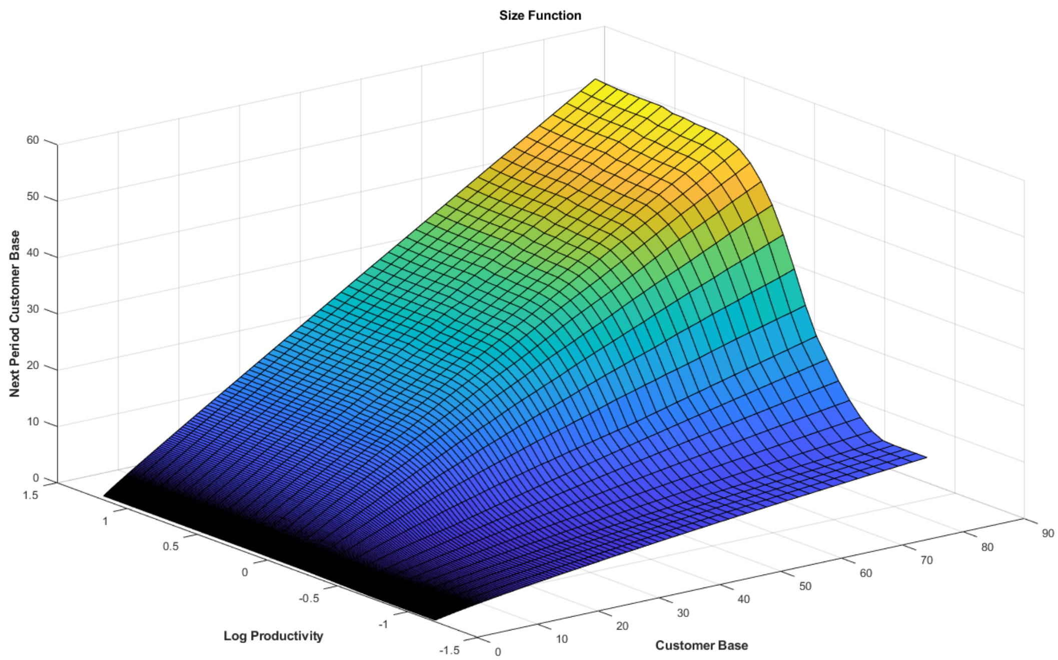


Figure A4: **Stationary Customer Base Decision Rule**



D Proofs

Proposition 1. *Suppose (1) $\theta \in (0,1)$, (2) $F(p)$ is differentiable and (3) $-\frac{pH_{pp}(m,z)}{H_p(m,z)} < 2$. Then $\exists \underline{\kappa} > 0$ such that $\forall \kappa > \underline{\kappa}$, the price function $p(m,z)$ is increasing in m as a solution to sellers' problem.*

Proof. First, the law of motion of customer base is rewritten as,

$$H(m,p) = mR(p) + h(m,\theta)N(p)$$

The derivatives are written in short forms in the rest of prove.

$$H_p = \frac{\partial H(m,z)}{\partial p}, \;\; H_{pp} = \frac{\partial^2 H(m,z)}{\partial^2 p}, \;\; H_m = \frac{\partial H(m,z)}{\partial m}$$

$$H_{mp} = \frac{\partial^2 H(m,z)}{\partial p \partial m}, \;\; \tilde{\pi}_p = \frac{\partial \tilde{\pi}(p,z)}{\partial p}, \;\; \tilde{\pi}_{pp} = \frac{\partial^2 \tilde{\pi}(p,z)}{\partial^2 p}$$

The main steps of the prove stated below.

Step 1. Show sellers' value function $W(m,z)$ is concave in m. This can be done by showing the period return function $H(m,p)\tilde{\pi}_p(p,z)$ is concave in m. Given $\theta < 1$, $h(m,\theta)$ is concave hence the return function is concave.

Step 2. Derive first order condition of sellers' problem, define it as an implicit function.

$$X(p,m,z) = \tilde{\pi}(p,z) + \frac{H(m,p)}{H_p(m,p)}\tilde{\pi}_p(p,z) + \beta E[W_1(H(m,p),z')|z] = 0$$

Step 3. Apply implicit function theorem and prove the signs of each partial derivative.

$$\frac{\partial X(p,m,z)}{\partial p} = \tilde{\pi}_p + \frac{H_p^2\tilde{\pi}_p + HH_p\tilde{\pi}_{pp} - HH_{pp}\tilde{\pi}_p}{H_p^2} + \beta E[W_{11}(H,z')|z]H_p$$

Arrange terms and use the closed form of $\tilde{\pi}(p,z)$ to substitute out $\tilde{\pi}_{pp}$,

$$\frac{\partial X(p,m,z)}{\partial p} = \frac{\tilde{\pi}_p}{H_p^2}\left[2H_p^2 - H\left(\frac{2}{p}H_p - H_{pp}\right)\right] + \beta E[W_{11}(H,z')|z]H_p > 0$$

The first term is positive from assumption (3) of the proposition. The second term is positive as $W(m,z)$ is concave in m and $H_p < 0$ (Lemma 1). Take partial derivative of $X(.)$ w.r.t m,

$$\frac{\partial X(p,m,z)}{\partial m} = \frac{H_m H_p - H H_{mp}}{H_p^2}\tilde{\pi}_p + \beta E[W_{11}(H,z')|z]H_m$$

Plug in the closed form of $H(m,p)$, arrange terms,

$$\frac{\partial X(p,m,z)}{\partial m} = \frac{(1-\theta)mh(m;\theta)(N'(p)R(p) - N(p)R'(p))}{H_p^2}\tilde{\pi}_p + \beta E[W_{11}(H,z')|z]H_m$$

The second term is negative because $W(m,z)$ is concave in m and $H_m > 0$. The sign of $N'(p)R(p) - N(p)R'(p)$ is negative following Lemma 2. Given $0 < \theta < 1$, we have $\frac{\partial X(p,m,z)}{\partial m} < 0$.

Lastly, apply implicit function theorem, $p(m,z)$ is the solution to $X(p,m,z) = 0$, it satisfy the following condition.

$$\frac{\partial p(m,z)}{\partial m} = -\frac{\partial X(p,m,z)}{\partial m} / \frac{\partial X(p,m,z)}{\partial p} > 0$$

□

Lemma 1 *Assume* $\gamma > 1$ *and* $\phi > 0$. *Then in any stationary equilibrium, shopping intensity* $s(p_1; wl + \pi, F)$ *satisfies the following condition, and is increasing in initial price and decreasing in income,*

$$s(p_1; wl + \pi, F) = \kappa^{-1/\phi}(wl+\pi)^{(1-\gamma)/\phi}\left(\int_{\underline{p}}^{p_1} F(p)p^{\gamma-2}dp\right)^{1/\phi}.$$

Proof. Conditional on the first stage price p_1, the problem of choose optimal shopping effort is stated as below.

$$\max_{s\in[0,1]} \left\{(1-s)u(p_1,s) + s\int u(min\{p_1,p_2\},s)dF(p_2)\right\}$$

where,

$$u(p,s)=\frac{c^{1-\gamma}}{1-\gamma}-\kappa\frac{s^{1+\phi}}{1+\phi},\ c=\frac{wl+\pi}{p}$$

Arrange the objective function,

$$\begin{aligned}
&(1-s)u(p_1,s)+s\int u(min\{p_1,p_2\},s)dF(p_2)\\
&=(1-sF(p_1))u(p_1,s)+s\int_{\underline{p}}^{p_1}u(p,s)dF(p)\\
&=(1-sF(p_1))u(p_1,s)+sF(p)u(p,s)\Big|_{\underline{p}}^{p_1}-s\int_{\underline{p}}^{p_1}F(p)du(p,s)\\
&=u(p_1,s)+s(wl+\pi)^{1-\gamma}\int_{\underline{p}}^{p_1}F(p)p^{\gamma-2}dp\\
&=\frac{(wl+\pi)^{1-\gamma}}{1-\gamma}p_1^{\gamma-1}-\kappa\frac{s^{1+\phi}}{1+\phi}+s(wl+\pi)^{1-\gamma}\int_{\underline{p}}^{p_1}F(p)p^{\gamma-2}dp
\end{aligned}$$

take the first order condition w.r.t. s, and rearrange,

$$s=\kappa^{-1/\phi}(wl+\pi)^{(1-\gamma)/\phi}\left(\int_{\underline{p}}^{p_1}F(p)p^{\gamma-2}dp\right)^{1/\phi}$$

The monotonicity claims holds by construction. □

Lemma 2. *Suppose $\theta\in(0,1)$, then $\exists\underline{\kappa}>0$ such that $\forall\kappa>\underline{\kappa}$, $-\frac{pN'(p)}{N(p)}>-\frac{pR'(p)}{R(p)}$ in any stationary equilibrium with a differentiable $F(p)$.*

Proof. Let the support of $F(p)$ be $[\underline{p},\bar{p}]$, plug in the closed form of $N(p)$ and $R(p)$,

$$-\frac{N'(p)}{N(p)}=\frac{s(p)\int\mathbf{1}_{\{p(\hat{m},z)=p\}}m\ d\lambda(m,z)}{\int\mathbf{1}_{\{p(\hat{m},z)>p\}}s(p(m,z))m\ d\lambda(m,z)}>\frac{s(p)\int\mathbf{1}_{\{p(\hat{m},z)=p\}}m\ d\lambda(m,z)}{s(\bar{p})\int\mathbf{1}_{\{p(\hat{m},z)>p\}}m\ d\lambda(m,z)}$$

$$-\frac{R'(p)}{R(p)}=\frac{s'(p)F(p)+s(p)f(p)}{1-s(p)F(p)}$$

Given the formula of $s(p)$ (Lemma 1),

$$s(p_1)=\kappa^{-1/\phi}((wl+\pi))^{(1-\gamma)/\phi}\left(\int_{\underline{p}}^{p_1}F(p)p^{\gamma-2}dp\right)^{1/\phi}$$

then $\forall p$ in the support of $F(p)$,

$$\lim_{\kappa\to\infty} s(p) = 0,\ \lim_{\kappa\to\infty} s'(p) = 0,\ \lim_{\kappa\to\infty} \frac{s(p)}{s(\bar{p})} > 0$$

Hence,

$$\lim_{\kappa\to\infty} -\frac{N'(p)}{N(p)} > 0 = \lim_{\kappa\to\infty} -\frac{R'(p)}{R(p)}$$

By the definition of limit, $\exists \underline{\kappa} > 0$ such that $\forall \kappa > \underline{\kappa}$, we have $-\frac{pN'(p)}{N(p)} > -\frac{pR'(p)}{R(p)}$. $\square$